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"Decomposing Firm Value"

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Abstract

What are the economic determinants of firms' market values? We answer this question through the lens of a generalized neoclassical model of investment with physical capital, quasi-fixed labor, and two types of intangible capital, knowledge capital and brand capital. We estimate the structural model using firm-level data on U.S. publicly traded firms and use the parameter values to infer the contribution of each input for explaining firms' market value in the last four decades. The model performs well in explaining both cross-sectional and time-series variation of firms' market values, with a time series R2 of 71% and a cross sectional R2 of 94%. We find that, on average, physical capital accounts for 29.6% of firms' market value, installed labor force accounts for 27.2%, knowledge capital accounts for 6.1%, and brand capital for 37.1%. These values vary substantially across industries and over time. We document that the importance of physical capital for firm value has decreased over time, while the importance of labor and brandcapital has increased, especially in industries that rely relatively more on high skill workers than on low skill workers. Overall, our value decomposition provides direct empirical evidence supporting models with multiple capital inputs as main sources of firm value.

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Decomposing Firm Value

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1 Introduction

Understanding the economic determinants of firm's market value is an important question that has attracted substantial research in economics. In this paper, we address this question through the lens of a generalized neoclassical model of investment with four different types of quasi-fixed inputs: physical capital (e.g. machines and plants), labor (the firm's installed labor force), and two types of intangible capital, namely knowledge capital (the firm's cumulated investment in innovation activities), and brand capital (the firm's cumulated investment in improving brand awareness). The rich model of the firm incorporates the evidence from Hall (2001), McGrattan and C. Prescott (2000), and Merz and Yashiv (2007) that, at the aggregate-level, intangible capital and the firm's installed labor force are important components of aggregate stock market values. Through structural estimation using data for a large cross section of publicly traded firms in the U.S. economy, we quantify the relative importance of the different capital inputs for understanding the level and the variation in the market values at the firm-level, both across industries and over time.

In the model, changing the quantity of the capital and labor inputs is costly, which we capture in a standard way through an adjustment cost function. For physical and intangible capital, these costs include planning and installation costs, costs related with production being temporarily interrupted, among other costs. For labor, these costs include the cost of hiring, and firing workers, as well as the cost of training new workers. Under constant returns to scale, a firm's valuation ratio (market value of equity plus debt divided by book value of the capital inputs) is directly linked to the shadow price of each installed capital/labor input and the capital/labor stocks of each input, and can be inferred in the data from investment and hiring data and the specification of an adjustment cost function. The basic intuition for this result follows from standard neoclassical theory of investment (Hayashi 1982). At the optimum, firms invest in each capital input until the marginal cost of an additional unit equals the present value of its future benefits, that is, the shadow price of the input. We use this result to compute the value of the installed capital and labor input as the product of the shadow price and the corresponding stock variable. The value of the firm is then the sum of the value of the capital and labor inputs.

In the presence of capital and labor adjustment costs, the market value of the installed stocks of these inputs is different from their book-values. In the case of labor, its book-value is zero because firms do not sell or buy workers as they do with capital goods, but its market-value might be different from zero if its costly to adjust the labor force. This is because, in equilibrium, firms extract rents from labor as a compensation for the costs of adjusting the labor force in the future. But if labor markets are competitive and frictionless (and there is no time-to-hire), labor inputs are paid their marginal product, and the rents from labor are zero. In this case, the contribution of the firms' installed labor force for the firm's market value is zero. The same logic applies to capital inputs, but here the book-value of these inputs is not zero, even without adjustment costs, because firms can sell (or buy) capital. For example, in the one-capital input model, without physical capital adjustment costs, the book-value of the firm is equal to the book-value of the physical capital stock.

Our estimation procedure is as follows. We estimate the structural parameters of the model by minimizing the distance between observed and model-implied valuation ratios (market value of equity and debt-to-book value of the capital stocks) as in Belo, Xue, and Zhang (2013) (henceforth BXZ), who in turn follow the original estimation approach in Liu, Whited, and Zhang (2009) (henceforth LWZ). To address the problem that measurement error in firm-level data can be large (e.g., Erickson and Whited 2000), we estimate the model parameters using portfolio-level moments, using several firm-level characteristics to sort firms into portfolios. We estimate the model across all firms in the economy, but also separately across industries. This allows us to characterize the importance of the different capital and labor inputs for firms' market value in different industries. Following Belo et al. (2017), we perform the estimation separately across low, medium, and high labor-skill industries. This industry classification is based on the fraction of workers in the industry that are high labor-skill workers.

We modify the estimation procedure in BXZ and LWZ in two important ways. First, to estimate the model parameters, we target cross-sectional portfolio-level moments that do not require aggregating the data to construct a portfolio-level aggregate valuation ratio. Specifically, for each portfolio, we target the cross-sectional median (computed across the firms in each portfolio) portfolio-level valuation ratio. This modification is important because, as show here, using artificial data, the parameter estimates obtained using the BXZ/LWZ portfolio-level aggregation procedure are subject to an aggregation bias, and hence do not have a structural interpretation. In contrast with this result, we show that the procedure proposed here allow us to recover the firm-level structural parameters of interest, which is crucial to provide a proper decomposition of the market value of the firm. As a robustness check, we show that the point estimates using our proposed procedure are similar to those obtained by targeting other sensible portfolio-level moments in the estimation, such as cross-sectional equal-weighted mean, inter-quartile range, and other moments of the cross sectional distribution of firm-level valuation ratios in each portfolio.

Second, we estimate the model parameters by minimizing the sum of squared difference (residuals) between the observed and model-implied moments of the valuation ratios of each portfolio in each year. Thus, our estimation procedure requires the model to match the realized time series of the observed valuation ratios as close as possible, not just the time-series means of the valuation ratios of each portfolio as in BXZ and LWZ. This is important in the context of our analysis because the contribution of each capital input for firm value is potentially changing over time.

To take the model to the data, we need to measure the knowledge and brand capital stocks. Given the intangible nature of these variables, the data for these intangible capital inputs is not readily available from firm's balance sheet data. Following previous studies, we construct firm-level measures of knowledge capital stock and brand capital stock from firm-level accounting data on research and development (R&D), and data on advertising expenses, respectively. Accordingly, we interpret R&D expenditures as a firms' investment to generate new (or improve current) ideas.

Similarly, we interpret advertising expenses as a firm's investment to enhance the value of brand names and brand awareness. We accumulate these expenditures using the perpetual inventory method to obtain the corresponding capital stocks.

Our main empirical findings can be summarized as follows. When the model is estimated across all firms in the economy, the parameter estimates imply that, on average, physical capital accounts for 29.6% of firms' market value, installed labor force accounts for 27.2%, knowledge capital accounts for 6.1%, and brand capital accounts for 37.1%. Thus, on average, the non physical capital inputs account for about 70% of the firm's market value.

The estimated relative importance of the capital and labor inputs for firms' market values varies across industries. On average, physical capital accounts for a large fraction of firm value in low labor-skill industries (about 60.5% of the firm's market value), but a significantly smaller fraction in high labor-skill industries (about 25% of the firm's market value). This result suggest that the standard one physical capital input model is a more appropriate model of the firm in low labor-skill industries. Related, we show that the average fraction of firm value attributed to labor and brand capital increases with the average labor-skill level of the industry. In the low labor-skill industry, the fraction of firm value that can be attributed to labor and brand capital is on average only 6.6% and 14%, respectively, whereas in the high labor-skill industry this fraction is 29.9% and 39% respectively. This result suggest that adding labor and brand capital to the one capital-input model is especially important for understanding the valuation of firm's in high labor-skill industries. Finally, we find that the average fraction of firm value attributed to knowledge capital decreases with the average labor-skill level of the industry.

What explains the estimated firm-value decomposition? We show that adjusting the four inputs in response to changing economic conditions is fairly costly. The parameter estimates imply that, consistent with Merz and Yashiv (2007), it is costly for a firm to adjust its labor force, especially in high labor-skill industries. Across all firms, we estimate that a firm' annual labor adjustment costs represent on average about 9.9% of total annual sales. This figure is significantly higher than the physical capital adjustment costs of about 0.6% of total annual sales. Similarly, our estimates show that it is costly to adjust both stocks of intangible capital. Knowledge capital adjustment costs are on average about 3.1% of total annual sales, and brand capital adjustment costs are on average 1.8% of total annual sales.

The estimated size of the adjustment costs of the different capital and labor inputs varies substantially across industries. This fact helps understand why the relative importance of the capital and labor inputs for firms' value also varies across industries. Labor adjustment costs increase significantly with the average labor-skill level of the industry. The fraction of sales lost due to labor adjustment costs are on average 11.5% in the high labor-skill industry but only 1.6% of firms' in the low labor-skill industry. Thus, consistent with previous studies, we find that it is more costly to replace high-skill than low-skill workers (see discussion in related literature section below). Similarly, knowledge capital adjustment costs increase with the average labor-skill level of the industry. The fraction of sales lost due to knowledge capital adjustment costs are on average 3.5% in the high labor-skill industry, and only 1% in the low labor-skill industry. This positive relationship between adjustment costs and average labor-skill of the industry is reversed for physical capital and brand capital. The fraction of sales lost due to physical capital adjustment costs are on average 0.5% in the high labor-skill industry, and 5.3% in the low labor-skill industry. Similarly, the fraction of sales lost due to brand capital adjustment costs are on average 1.2% in the high labor-skill industry, and 7.6% in the low labor-skill industry.

In terms of model fit, the model performs well in explaining both time-series and cross-sectional variation of valuation ratios across several portfolio sorts, with a time series R^2 of 71% and a cross sectional R^2 of 94%, when estimated across all firms in the economy. To help understand this good fit of the model and the relative importance of each capital input for firm's valuation, we estimate restricted versions of the model using subsets of the capital and labor inputs. Consistent with BXZ, we find that the standard one-physical capital input model does a reasonable job explaining the cross-sectional variation in the average valuation ratio across portfolios with a high cross-sectional

R^2 of 64%. But the one-capital input model fails in explaining the time-series variation in the valuation ratios. The time-series R^2 of the one-capital input model is only 9%, versus 71% in the baseline model. Thus, we conclude that the benefit of incorporating additional quasi-fixed inputs in the neoclassical investment model comes primarily from improving the model's ability to capture the time-series variation in the valuation ratios. Across industries, the model performs particularly well in explaining the time-series and cross-sectional variation in the high labor-skill industry, with a time-series R^2 of 67% and a cross sectional R^2 of 94%. The model fit in low skill industry is more modest but still reasonable, with a time-series R^2 of 38% and a cross sectional R^2 of 74%.

The comparison of the model fit across the different (restricted) specifications of the model further shows that adding labor to the standard one-capital input model has a first order impact on the model's fit. When quasi-fixed labor is added to the one-capital input model, the time-series R^2 increases from 9% to 53%. Adding brand capital also has an important impact on the model's fit, although slightly smaller than the impact of labor. When brand capital is added to the one-capital input model the time-series R^2 increases from 9% to 46%. The improvement in the model's fit due to the addition of knowledge capital is more modest. In this case, the time-series R^2 increases only slightly, from 9% to 14%.

We also investigate the time-series of the value, and value-shares, of each input. Across all firms, the importance of physical capital input has decreased over our sample period from 48.6% in the 70s to 18.2% in the 10s. In the opposite direction, the importance of labor input and brand capital for firm value has increased over our sample period. The contribution of labor for firm value increased from 20.4% in the 70s, to 36.4% in the 10s, while the contribution of brand capital for firm value increased from 23.3% in the 70s, to 41.4% in the 10s. The contribution of knowledge capital for firm value was relatively small during the entire period, although is has slightly decreased over the sample period. Even tough the increase in the importance of labor and brand capital, and the corresponding decrease of physical capital for firm value is pervasive across all industries, the trends are significantly more pronounced in high labor-skill industries. Taken together, our results highlights the importance of labor and brand capital for understanding firm value, especially in the recent decades, and in industries that rely relatively more on workers with high average labor-skill levels.

Finally, we also investigate the risk properties of the capital and labor inputs by examining the correlation of the cyclical components of the value, and firm-value shares, of each input with the cyclical component of aggregate sales, and also the volatility of each input value. We find that while the share of labor on firms' market value is procyclical, the shares of the other capital inputs is countercyclical, especially in high labor-skill industries. Thus, the importance of labor input for understanding firm's value is higher during good economic times. In addition, we find that, across all firms, all input values are procyclical, especially labor and knowledge capital inputs. The value of labor is also the most volatile component of firm value. These findings suggest that understanding the dynamics of labor inputs over time and across firms seems important for understanding the dynamics of firm's market value in financial markets.

Related Literature

Our work is related to the large literature on valuation.[1] Our approach is closely related to the supply approach to valuation developed in BXZ, extended to a setup in which multiple and heterogeneous capital and labor inputs, not just physical capital, can contribute to the firm's market value. As we show here, even tough the baseline one capital-input model does a good job capturing the cross-sectional variation in valuation ratios across portfolios, it is unable to explain its time-series variation. Importantly, our modified estimation method allows us to recover the firm-level structural parameters, which are crucial to perform valuation at the firm- not just portfolio-level, thus substantially increasing the usefulness of the approach in practice. In a recent study, Gonçalves, Xue, and Zhang (2017) also address the aggregation issues in the original LWZ portfolio-level aggregation approach (see also Zhang 2017 for a discussion of the aggregation bias in the standard tests of the investment-based model). Using a variation of one of the alternative

[1]See BXZ for an overview of the valuation literature in both Finance, Economics, and Accounting.

estimation methods proposed here (target the portfolio-level cross-sectional mean), they show that the baseline investment-based model can simultaneously capture the variation in average returns across a large set of portfolios (value, momentum, profitability), and other empirical patterns in the cross section, with a stable set of parameter values, in contrast with the results obtained using the original portfolio-level aggregation procedure proposed in LWZ.

Our paper is related to the asset pricing literature on intangible capital and firm risk. Eisfeldt and Papanikolaou (2013) estimate the value of organization capital using a model of the sharing rule between a firm's owners and its key talent. Eisfeldt and Papanikolaou (2013) show that firms with more organization capital are riskier than firms with less organization capital. Following Lev and Radhakrishnan (2005), the authors measure organization capital from selling, general and administrative expenses (SG&A). Thus, this measure of organization capital is a broad concept: it includes the value of the labor force, knowledge capital, brand capital, among other. Because our goal is to decompose the value of the firm and understand the relative contribution of labor and the different intangible capital inputs for firms' market value, we do not use this broad measure, and instead focus on measures of the separate components. Hansen, Heaton, and Li (2012) study the risk characteristics of intangible capital. Li and Liu (2012) and Vitorino (2014) study the importance of intangible capital in a q-theory model via structural estimation. We build on their work by considering a general model that includes both knowledge and brand capital, and most importantly also frictions in the labor inputs. Hence, we provide a more accurate assessment of the contribution of each capital input to firm value, and investigate their business cycle properties.

A growing literature has further shown the importance of intangible capital for corporate decisions. Falato, Kadyrzhanova, and Sim (2014), building on earlier work by Carol A. Corrado and Sichel (2009) and Corrado and Hulten (2010), show that intangible capital is the most important firm-level determinant of corporate cash holdings, with the rise in intangible capital being a fundamental driver of the secular trend in US corporate cash holdings over the last decades. We differ from these studies because our structural model allows us to measure the market value of the capital inputs, not just the book-values. As we show here, a firm-value decomposition based on book-value of the capital inputs is significantly different from a firm-value decomposition based on the market value of the inputs. Peters and Taylor (2017) propose a new proxy for Tobin's Q that accounts for intangible capital, and show that it is a superior proxy for explaining total firm investment in physical and intangible capital. Our structural model of the firm, which also incorporates intangible capital, provides a quantitative decomposition of Tobin's Q into the value of each capital input according to the optimal corporate policies including labor hiring, and investment in physical and intangible capital.

The findings in our paper are also related to the large literature that tries to understand the trend in the labor share in the economy. The increase in the importance of labor for firm value that we document here resembles the evidence in Hartman-Glaser, Lustig, and Xiaolan (2017) who show that the cross-sectional average labor share of publicly traded firms has increased over time in the U.S. economy (in contrast with the well documented decrease of the aggregate labor share over the same sample period, as noted in Elsby, Hobijn, and Şahin 2013, Karabarbounis and Neiman 2013, among others). The difference is that we compute the importance of the value of labor for firm-value, not for value added as in Hartman-Glaser, Lustig, and Xiaolan (2017).

An important strand of the asset pricing literature already documents the effect of labor market frictions on stock returns and firm value.[2] The theoretical approach in this paper is related to the work of Merz and Yashiv (2007), who builds upon earlier work of Cochrane (1991). Merz and Yashiv (2007) consider an aggregate representative firm facing adjustment costs in both capital and labor, and focus on the estimation of the production and adjustment cost functions. They show that adding labor adjustment costs substantially improves the model's ability to capture the dynamics of aggregate stock market value. We build on the Merz and Yashiv (2007)'s setup by including two additional types of costly intangible capital. Extending the model to the firm-level

[2]A partial list of studies linking labor market variables to asset prices include Mayers (1972), Fama and Schwert (1977), Campbell (1996), Jagannathan and Wang (1996), Jagannathan, Kubota, and Takehara (1998), Santos and Veronesi (2005), Boyd, Hu, and Jagannathan (2005), and Lustig and Van Nieuwerburgh (2008). The interpretation of the empirical facts in these studies is silent about the production-side of the economy (technology).

further allows us to exploit not only time-series data, but also firm-level cross-sectional data. Belo, Lin, and Bazdresch (2014) add labor adjustment costs into Zhang (2005)'s model and show that labor hiring negatively predicts future returns in the cross section both in model simulations and in the data. In our work, we focus on equity valuation ratios and we provide a structural estimation of the frictions in (physical and intangible) capital and labor markets.

Our work is also related to the large literature on labor demand and capital investment which investigates the importance of capital and labor adjustment costs to explain investment and hiring dynamics.[3] The estimated economic magnitude of adjustment costs is still subject to debate. For example, Shapiro (1986) shows that large estimates of labor adjustment costs are important to match investment and hiring dynamics, particularly for non production workers. Hall (2004), however, estimates both capital and labor adjustment costs to be negligible at the two-digit SIC industry level. We add to this literature by providing structural estimates of adjustment costs for multiples types of capital and labor inputs based on financial market data.

Our paper also contributes to the literature on the importance of capital heterogeneity. For example, using a dataset of Japanese firms, Hayashi and Inoue (1991) find strong empirical support for the relationship between aggregate capital growth and Tobin's Q derived in a model with multiple capital goods. Similarly, Chirinko (1993) estimates an investment model with multiple capital inputs and adjustment technologies, and find significant evidence in favor of capital heterogeneity. Our firm value decomposition provides additional direct empirical evidence supporting models with multiple capital inputs.

[3] See, for example, on capital: Cooper and Haltiwanger (1997), Caballero et al. (1995), Cooper, Haltiwanger, and Power (1999) and Cooper and Haltiwanger (2006); on labor: Hamermesh (1989), Bentolila and Bertola (1990), Davis and Haltiwanger (1992), Caballero and Engel (1993), Caballero, Engel, and Haltiwanger (1997), Cooper, Haltiwanger, and Willis (2015); on joint estimation of capital and labor adjustment costs Shapiro (1986), Galeotti and Schiantarelli (1991), Hall (2004), Merz and Yashiv (2007) and Bloom (2009). Bond and Van Reenen (2007) survey the literature and Hamermesh (1996) reviews a set of direct estimates of the labor adjustment costs.

2 The Model of the Firm

This section solves the optimal production decision of a firm. The model is a neoclassical model of the firm as in LWZ/BXZ (we use their notation whenever possible), extended to a setup with several quasi-fixed inputs. Time is discrete and the horizon infinite. Firms choose costlessly adjustable inputs each period, while taking their prices as given, to maximize operating profits (revenues minus the expenditures on these inputs). Taking these operating profits as given, firms optimally choose the physical and intangible capital investment, hiring, and debt to maximize the market value of equity. To save notation, we denote the firm's i set of capital (and labor) inputs as $\mathbf{K}_{i,t}$. This set includes the physical capital stock ($K^P_{i,t}$), labor stock ($L_{i,t}$), knowledge capital stock (an intangible, and hence unmeasured (U) capital input in firm's accounts, U^K_{it}), and brand capital stock (another intangible capital input, U^B_{it}). Similarly, we denote the firm's i set of investment in the capital inputs as $\mathbf{I}_{i,t}$. This set includes the investment in physical capital ($I^P_{i,t}$), investment in labor stock, that is, gross hiring ($H_{i,t}$), investment in knowledge capital (I^K_{it}), and investment in brand capital I^B_{it}.

2.1 Technology

The operating profits function for firm i at time t is $\Pi_{it} \equiv \Pi(\mathbf{K}_{it}, X_{it})$, in which X_{it} denotes a vector of exogenous aggregate and firm-specific shocks. We assume that the firm has a production function with constant returns to scale.

The law of motion of the firm's capital inputs and labor force are given by:

$$K^P_{it+1} = I^P_{it} + (1 - \delta^P_{it})K^P_{it} \tag{1}$$

$$L_{it+1} = H_{it} + (1 - \delta^L_{it})L_{it} \tag{2}$$

$$U^K_{it+1} = I^K_{it} + (1 - \delta^K_{it})U^K_{it} \tag{3}$$

$$U^B_{it+1} = I^B_{it} + (1 - \delta^B_{it})U^B_{it}, \tag{4}$$

where δ_{it}^P, δ_{it}^K and δ_{it}^B are the exogenous depreciation rates of physical, knowledge and brand capital, respectively. δ_{it}^L is the employee quit rate, the rate at which the worker's leave the firm for voluntary reasons.

Firms incur adjustment costs when investing. The augmented adjustment costs function, denoted $C_{it} \equiv C(\mathbf{I}_{i,t}, \mathbf{K}_{i,t})$, is increasing and convex in investment/hiring, decreasing in the capital stocks, and has constant returns to scale.

2.2 Taxable Profits and Firms' Payouts

We allow firms to finance investments with debt. At the beginning of time t, firm i issues amount of debt, denoted B_{it+1}, which must be repaid at the beginning of time $t+1$. Let r_{it}^B denote the gross corporate bond return on B_{it}. We can write taxable corporate profits as operating profits minus depreciation, adjustment costs, and interest expense:

$$\Pi_{it} - I_{it}^K - I_{it}^B - W_{it}L_{it} - \delta_{it}^K K_{it} - C_{it} - (r_{it}^B - 1)B_{it}.$$

Let τ_{it} be the corporate tax rate. We define the payout of firm i as:

$$D_{it} \equiv (1-\tau_t)[\Pi_{it} - C_{it} - I_{it}^K - I_{it}^B - W_{it}L_{it}] - I_{it}^P + B_{it+1} - r_{it}^B B_{it} + \tau_t \delta_{it}^K K_{it}^P + \tau_t (r_{it}^B - 1)B_{it}, \quad (5)$$

in which $\tau_t \delta_{it}^K K_{it}^P$ is the depreciation tax shield and $\tau_t(r_{it}^B - 1)B_{it}$ is the interest tax shield. Adjustment costs are expensed, consistent with treating them as foregone operating profits.

2.3 Equity Value

Firm i takes the stochastic discount factor, denoted M_{t+1}, from period t to $t+1$ as given when maximizing its cum-dividend market value of equity:

$$V_{it} \equiv \max_{\{\mathbf{I}_{it+\triangle t}, \mathbf{K}_{it+\triangle t+1}, B_{it+\triangle t+1}\}_{\triangle t=0}^{\infty}} E_t\left[\sum_{\triangle t=0}^{\infty} M_{t+\triangle t} D_{it+\triangle t}\right], \quad (6)$$

subject to a transversality condition given by $\lim_{T\to\infty} E_t[M_{t+T}B_{it+T+1}] = 0$.

Let $P_{it} \equiv V_{it} - D_{it}$ be the ex-dividend equity value. Appendix A shows that firms' value maximization implies that:

$$P_{it} + B_{it+1} = q_{it}^P K_{it+1}^P + q_{it}^L L_{it+1} + q_{it}^K K_{it+1}^K + q_{it}^B K_{it+1}^B, \tag{7}$$

in which

$$\begin{aligned} q_{it}^P &\equiv 1 + (1-\tau_t)\partial C_{it}/\partial I_{it}^P & (8) \\ q_{it}^L &\equiv (1-\tau_t)\partial C_{it}/\partial H_{it} & (9) \\ q_{it}^K &\equiv (1-\tau_t)\left[1 + \partial C_{it}/\partial I_{it}^K\right] & (10) \\ q_{it}^B &\equiv (1-\tau_t)\left[1 + \partial C_{it}/\partial I_{it}^B\right] & (11) \end{aligned}$$

The variables $q_{it}^P, q_{it}^L,$ q_{it}^K and q_{it}^B measure the shadow prices of physical capital, labor force, knowledge capital, and brand capital, respectively.

Equation (7) provides a formula to decompose the firm value as the sum of the value of the firm's installed labor and capital inputs. Specifically, the fraction of firm value that is attributed to these inputs is as follows:

$$\begin{aligned} \mu_{it}^P &= \frac{q_{it}^P K_{it+1}^P}{q_{it}^P K_{it+1}^P + q_{it}^L L_{it+1} + q_{it}^K U_{it+1}^K + q_{it}^B U_{it+1}^B} & (12) \\ \mu_{it}^L &= \frac{q_{it}^L L_{it+1}}{q_{it}^P K_{it+1}^P + q_{it}^L L_{it+1} + q_{it}^K U_{it+1}^K + q_{it}^B U_{it+1}^B} & (13) \\ \mu_{it}^K &= \frac{q_{it}^K K_{it+1}^K}{q_{it}^P K_{it+1}^P + q_{it}^L L_{it+1} + q_{it}^K U_{it+1}^K + q_{it}^B U_{it+1}^B} & (14) \\ \mu_{it}^B &= \frac{q_{it}^B K_{it+1}^B}{q_{it}^P K_{it+1}^P + q_{it}^L L_{it+1} + q_{it}^K U_{it+1}^K + q_{it}^B U_{it+1}^B} & (15) \end{aligned}$$

A fundamental goal of the empirical analysis is to characterize these weights including their variation over time and across industries.

3 Econometric Methodology

In this section we specify the functional forms and the estimation method used to obtain the structural parameters. In addition, we describe the data and report descriptive statistics for the key variables of the model.

3.1 Functional Forms

We consider the following flexibile functional form for the adjustment cost function:

$$C_{it} = \frac{1}{\nu_P}\left|\theta_P \frac{I_{it}^P}{K_{it}^P}\right|^{\nu_P} K_{it}^P + \frac{1}{\nu_L}\left|\theta_L \frac{H_{it}}{L_{it}}\right|^{\nu_L} W_{it}L_{it} + \frac{1}{\nu_K}\left|\theta_K \frac{I_{it}^K}{U_{it}^K}\right|^{\nu_K} U_{it}^K + \frac{1}{\nu_B}\left|\theta_B \frac{I_{it}^B}{U_{it}^B}\right|^{\nu_B} U_{it}^B, \quad (16)$$

in which W_{it} is the wage rate, which the firm takes as given, $\theta_P, \theta_L, \theta_K, \theta_B > 0$ are the slope adjustment cost parameters, and $\nu_P, \nu_L, \nu_K, \nu_B > 1$ are the curvature adjustment cost parameters. Labor adjustment costs are proportional to the firms' wage bill, as in Bloom (2009). This helps to make the units of labor adjustment costs (measured in number of workers) similar to the other capital inputs which are measured in (real) dollar values, an adjustment that is important for the empirical results below. This specification nests the standard quadratic functional forms as special cases when the curvature parameters are equal to two.[4]

The absolute value specification of the adjustment cost function allows for negative investment rates and improves the stability of the estimation of curvature parameters.[5] This functional form generalizes the one-capital input functional form specification used in BXZ to multiple inputs.[6]

The adjustment cost function in equation (16) implies that the shadow prices of the capital

[4]We place the slope adjustment cost parameters inside the absolute values of the equation (16) to make the units of the slope adjustment cost parameters independent of the curvature parameter. This improves identification and stability during the estimation. See Belo, Xue, and Zhang (2013)for a similar approach.

[5]When the curvature parameters are greater than one, $v_i > 1$, this function is continuous along its entire domain including at zero since left and right derivatives at zero coincide. See also Kogan and Papanikolaou (2012) for a similar specification in the context of one capital input model.

[6]Although not explicitly stated, BXZ also use the absolute value function to deal with negative investment rates observed in the data.

inputs are given by:

$$q_{it}^{P} \equiv 1+(1-\tau_t)\theta_P^{\nu_P}\left|\frac{I_{it}^{P}}{K_{it}^{P}}\right|^{\nu_K-1}\text{sign}\left(\frac{I_{it}^{P}}{K_{it}}\right) \tag{17}$$

$$q_{it}^{L} \equiv (1-\tau_t)\theta_N^{\nu_N}\left|\frac{H_{it}}{L_{it}}\right|^{\nu_N-1}\text{sign}\left(\frac{H_{it}}{L_{it}}\right)W_{it} \tag{18}$$

$$q_{it}^{K} \equiv (1-\tau_t)\left[1+\theta_U^{\nu_K}\left|\frac{I_{it}^{K}}{U_{it}^{K}}\right|^{\nu_K-1}\text{sign}\left(\frac{I_{it}^{K}}{U_{it}^{K}}\right)\right] \tag{19}$$

$$q_{it}^{B} \equiv (1-\tau_t)\left[1+\theta_U^{\nu_B}\left|\frac{I_{it}^{B}}{U_{it}^{B}}\right|^{\nu_B-1}\text{sign}\left(\frac{I_{it}^{B}}{U_{it}^{B}}\right)\right]. \tag{20}$$

We use the sign function to express the equilibrium shadow prices of each capital inputs in a compact manner, that is, using one equation, instead of a piecewise function. This is because, given the absolute value specification, the sign associated with the investment and hiring rate term switches depending on whether the input-specific investment or hiring rate is positive or negative.

3.2 Estimation Procedure

Equation (7) links firm value to the value of its labor and capital inputs. Since firm values are not necessarily stationary, it is useful to scale this variable for estimation purposes. We divide both sides of equation (7) by the sum of the firm's capital inputs (not including labor), which we denote as A_{it+1}, a measure of the firm's total (effective) assets $A_{it+1} \equiv K_{it+1}^{P} + U_{it+1}^{K} + U_{it+1}^{B}$. We do not include labor inputs to compute total assets because labor is measured in different units (number of workers as opposed to dollars in real terms). Hence, we write a firm's valuation ratio ($VR_{it} \equiv (P_{it} + B_{it+1})/A_{it+1}$) as:

$$VR_{it} = q_{it}^{P}\frac{K_{it+1}^{P}}{A_{it+1}} + q_{it}^{L}\frac{L_{it+1}}{A_{it+1}} + q_{it}^{K}\frac{K_{it+1}^{K}}{A_{it+1}} + q_{it}^{B}\frac{K_{it+1}^{B}}{A_{it+1}}. \tag{21}$$

The left-hand side (LHS) of the equation can be directly measured in the data from equity price data and debt data (and measures of the capital stocks, which we discuss below). The right hand side (RHS) of the equation is the predicted valuation ratio from the model, $\widehat{VR}_{it}$, which depends on the model parameters.

Aggregation Issues

Equation (21) establishes an exact relationship between the firm's observed valuation ratio and the model-implied valuation ratio. Since measurement error in firm-level data is likely to be large (e.g., Erickson and Whited 2000), and large outliers can reduce the ability to recover structural parameters, we perform the estimation at the portfolio-level as in BXZ, which in turn follow the original approach in LWZ. The use of portfolio-level data has several appealing features. First, the focus on portfolio-level moments allows us to reduce the noise in the firm-level data. In addition, the portfolio-level moments are less sensitive to firm entry and exit, and less affected by missing observations as the firm-level regressions. This is an important consideration in the context of our application because the R&D and Advertising expenses data necessary to construct the knowledge capital and brand capital stocks are missing for a nontrivial fraction of the firms in Compustat (as described in Section 3.3 below). Unlike LWZ/BXZ, however, we estimate the model parameters by targeting cross-sectional portfolio-level moments that do not require aggregating the data to construct a portfolio-level aggregate valuation ratio, thus allowing us to recover the firm-level structural parameters.

Aggregation in LWZ Before explaining our estimation method, it is useful to revisit the aggregation procedure in LWZ/BXZ because it is the limitation of the procedure in the context of our research question that justifies the use of an alternative method.[7] Following the approach in LWZ/BXZ, one would estimate the valuation equation at the portfolio-level by first computing the portfolio-level characteristics (e.g. portfolio-level investment rates), and then plugging these characteristics directly in the valuation equation (21) to obtain the observed and the model-implied valuation ratios. Specifically, in year t, the portfolio j investment rate in physical capital is

[7] Liu, Whited, and Zhang (2009) estimate the model predicted investment returns rather than valuation ratios using portfolio-level aggregated data. The two are closely related, however, because to a first order approximation, the investment return is the valuation equation in first differences.

computed as:

$$\frac{I_{jt}^{K}}{K_{jt}} = \frac{\sum_i I_{j,i,t}^{K}}{\sum_i K_{j,i,t}}, \text{ i} \in \text{Portfolio j} \tag{22}$$

which is then substituted in equation (17) to obtain the portfolio-level shadow price of the physical capital stock. Similarly, the portfolio level observed valuation ratio and capital stocks are given by:

$$\begin{aligned} VR_{jt} &= \frac{\sum_i (P_{it} + B_{it+1})}{\sum_i A_{it}} \\ K_{jt} &= \sum_i K_{j,i,t} \text{ , i} \in \text{Portfolio j.} \end{aligned}$$

The estimation would then proceed to estimate the parameter values by the Generalized Method of Moments (GMM) under the identification assumption that the model errors, computed as the difference between the portfolio-level aggregated observed and model-implied valuation ratios are on average zero.

The LWZ/BXC approach provides a powerful framework for identifying robust links between valuation ratios/stock returns and portfolio-level characteristics. In addition, this approach averages out measurement error in firm-level data in a convenient and elegant manner. Unfortunately, the aggregation procedure complicates the interpretation of the parameter estimates. Specifically, by using the portfolio-level characteristics computed as in equation (22) to construct the shadow price of the capital input in equations (17), the procedure does not guarantee the recovery of the true firm-level structural parameters because the shadow prices of the capital inputs are, in general, nonlinear functions of the characteristics. Appendix B provides a more detailed analysis of this issue and provides estimates of the aggregation bias for a particular calibration of the adjustment cost function in a one capital input model.[8]

Our Alternative Estimation Procedure To recover the firm-level structural parameters we thus modify the econometric approach proposed in LWZ. As noted, in theory, any moment of the observed firm-level valuation ratios in equation (21) should be equal to any corresponding

[8]Belo and Deng (2018) provide a general analysis of aggregation and other economic issues in the context of empirical tests of investment-based models.

moment of the model-implied firm-level valuation ratios. Thus, we target cross-sectional portfolio-level moments that do not require aggregating the data to construct a portfolio-level aggregate valuation ratio, hence avoiding the aggregation bias. Specifically, in each year, we compute the portfolio-level valuation ratio by taking the cross-sectional median of the firm-level observed and model-implied valuation ratios, which we refer to as cross-sectional median (XSMED) estimation. Since the median is insensitive to outliers, it is a natural moment to use in the estimation to mitigate the influence of large outliers in firm-level data. The median is also better suited to describe the economic behavior of the typical firm in the economy, and hence provide a better link to the model of the firm used here.[9] We use this estimation procedure to produce the baseline results.

We perform the estimation of the valuation equation (21) under the standard assumption that the portfolio-level valuation ratio moments (which, in the baseline specification, is the median valuation ratio across all firms in the portfolios) are observed with error by the econometrician:

$$VR_{it}^{MOM} = \hat{VR}_{it}^{MOM}(\Theta) + \varepsilon_{it} \tag{23}$$

where $\hat{VR}_{it}^{MOM}(\Theta)$ denotes the model-implied portfolio-level moment (MOM) of the cross-section of firm-level valuation ratios of the firms in portfolio i at time t, Θ represents the vector of structural parameters including an intercept, $\Theta = [\theta_P, \theta_L, \theta_K, \theta_B, \nu_P, \nu_L, \nu_K, \nu_B, \alpha]$, and ε captures measurement error in portfolio-level moment.[10] The parameter α is an intercept that we include in the estimation to allow for average nonzero measurement error. Based on equation (23), we estimate the model parameters by nonlinear least squares (NLLS), that is, we minimize the distance between the portfolio-level observed and model-implied valuation ratios moments:

$$\widehat{\Theta} = \arg\min_{\Theta} \frac{1}{TN} \sum_{t=1}^{T} \sum_{j=1}^{N} \left(VR_{jt}^{MOM} - \hat{VR}_{jt}^{MOM}(\Theta) \right)^2 .$$

Thus, unlike LWZ and BXZ, who estimate the model parameters by matching the time series

[9] Our relatively simple model is less appropriate for the valuation of the superstar firms like Apple or Facebook.

[10] Mismeasured components of the valuation ratio such as the market value of debt and the capital inputs can be better observed by firms than by econometricians. Furthermore, the intrinsic value of equity can temporarily diverge from the market value of equity.

means of the observed and model-implied portfolio valuation ratios, the use of NLLS in our estimation requires the model to match the realized time series of the observed valuation ratios as close as possible. We then compute bootstrapped standard errors that is robust to cross-sectional and time-series correlation.[11]

As a robustness check, we also consider an alternative estimation approach which targets other sensible portfolio-level moments in the estimation, such as cross-sectional equal-weighted mean (XSEW). In Appendix B we show that, under the assumptions described here, the baseline approach (XSMED) and this approach (XSEW) recover the underlying firm-level structural parameters. In addition, as an additional robustness check reported below, we further investigate if the parameter estimates vary significantly from the baseline estimation if we target other portfolio-level moments, such as inter-quartile range, and the 25^{th} and 75^{th} percentiles (not just the 50^{th} percentile as in the baseline approach) of the cross sectional distribution of the firm-level valuation ratios in each portfolio.

3.3 Data and Test Assets

Sample selection: Our sample consists of all common stocks on NYSE, Amex, and Nasdaq from 1975 to 2013. The firm-level data are from the Center for Research in Security Prices (CRSP)/Compustat Merged (CCM) – Fundamentals Annual database. We only include firms with fiscal year ending in the second half of the calendar year.

We remove duplicate observations resulting from a fiscal year-end change. CCM has several observations that are duplicates as a result of a firm having multiple securities issued with different PERMNO. These securities often have different share classes in the CRSP dataset. However, it is important to use all of these securities' market values in calculating the firm market value. We flag these duplicates and sum their securities' market values to get a total firm market value, then drop the duplicate observations. We also only consider common stocks (CRSP share code 10 and 11) and firms trading on the NYSE, AMEX, and NASDAQ exchanges (CRSP exchange code 1, 2, and

[11] We calculate bootstrapped standard errors using 20% of the sample with replacement.

3). We also keep only US firms (FIC = "USA") and drop firms using Canadian dollars (CURCD = "CAD").

Firms sometimes drop out of the Compustat sample and reappear later. This creates a gap in the data that can be treated in one of two ways: we can estimate the data during the gap, or we can treat it as if a new company enters the sample. When constructing measures of capital stock (as described below), we estimate the initial capital stock from the first investment observation. It is preferable to maintain the capital stock across the data gap rather than treat the gap as if it creates a new firm. Therefore, we mark these gaps in the data. We interpolate missing values across the gap and use the interpolated values to keep track of capital stocks through the gap.

Physical capital data: The initial physical capital stock, K_{it}^{P}, is given by net property, plant, and equipment (Compustat data item PPENT), and investment in physical capital, I_{it}^{P}, is given by capital expenditures (item CAPX) minus sales of property, plant, and equipment (item SPPE). We set SPPE to zero if missing. The capital depreciation rate, δ_{it}^{K}, is the amount of depreciation (item DP) divided by the capital stock. We construct an investment-price adjusted capital stock that accounts for changes in the real cost of physical capital investment by repricing last period's capital stock using today's price of investment: $K_{t+1}^{P} = K_{t}^{P}(1-\delta_t)\frac{P_{t+1}}{P_t} + I_{t+1}$. The price of investment is the BEA price index for non-residential fixed investment, NIPA Table 1.1.9, line 9.

Labor data: The labor stock, L_{it}, is number of employees (item EMP). The labor market data on wage rate and labor separation rate is not available at the firm level (the firm level wage bill data in COMPUSTAT is missing for more than 80% of the firms on our sample). We measure these variables at the industry level as follows:

Wage rate per worker: We measure W_{it} using annual data from the Bureau of Economic Analysis (BEA), National Income and Product Accounts (NIPA), Section 6. We compute the industry level (annual) wage rate per worker as the ratio of the total compensation of employees (which includes wage and salary accruals and supplements to wages and salaries) to the total number of employees in the industry. We use compensation of employees by industry from

Tables 6.2B-D and full-time and part-time employees by industry from Tables 6.4B-D. The data is aggregated at a fairly high level. Industries are defined using SIC prior to 1999 and using NAICS after. The BEA provides tables that link SIC and NAICS codes to the line items reported in the NIPA tables. This usually involves linking at the two- or three-digit level for SIC codes, and two-, three-, and four-digit levels for NAICS. We use the links found on the BEA website (http://www.bea.gov/industry/gdpbyind_data.htm), as well as in Yuskavage 2007).

Employee quit rate: We measure annual employee quit rate δ_{it}^L using data for 16 major industry groups based on NAICS codes from the Job Openings and Labor Turnover Survey (JOLTS) available from the Bureau of Labor Statistics (BLS). Because this data is only available since 2001, we proceed as follows. We estimate a time-varying quit rate by regressing, for each major industry group in JOLTS, the industry level quit rates on real GDP growth, unemployment, the labor vacancy rate, and a measure of labor market tightness. For each industry, we then extend the quit rate back to cover the entire sample. We also use the same procedure to estimate the time-varying aggregate JOLTS quit rate, and assign this rate to industries not covered in JOLTS or with missing industry code. This procedure allows us to have both cross-sectional and time-varying variation in the employee quit rate.

Knowledge capital data: Following Falato, Kadyrzhanova, and Sim (2014) we construct the firm's stock of knowledge capital from past expenditures data on research and development (R&D) (item XRD) and using the perpetual inventory model:[12]

$$U_{t+1}^K = U_t^K(1-\delta^K)\frac{P_{t+1}^K}{P_t^K} + I_{t+1}^K \tag{24}$$

P_t^K is the BEA price index for intellectual property products, R&D, from the Federal Reserve Economic Data (FRED) database. To implement the law of motion in equation (24) we must choose an initial stock and a depreciation rate. Using the perpetual inventory method, we choose

[12]See also Lev and Radhakrishnan (2005), Eisfeldt and Papanikolaou (2013), and Li and Liu (2012) for similar applications. The Bureau of Economic Analysis uses a similar methodology to construct a stock of Research and Development capital, see Sliker (2007).

the initial stock according to:

$$U_0^K = \frac{I_0^K}{g^K + \delta^K}.$$

in which I_0^K is the firms' investment in knowledge capital in the first year in the sample. We choose g^K to match the average growth rate of R&D investments, which in our sample is about 10%. We use the industry-specific depreciation rates estimated in Li (2012) for 10 industries, and use 15% for the remaining firms following Peters and Taylor (2017). Once we have the initial capital stock, we iterate forward using the appropriate depreciation rate, R&D investment amount, and investment price index. The investment rate on knowledge capital is then given by the ratio of the current period investment and the beginning of the period corresponding knowledge capital stock I_t^K/K_t^K.

Brand capital data: The construction of the brand capital stock is analogous to the construction of the knowledge capital stock. Following Belo, Lin, and Vitorino (2014) we construct the firm's stock of the brand capital from past expenditures data on advertising expenses (item XAD) and using the perpetual inventory model:

$$U_{t+1}^B = U_t^B(1-\delta^B)\frac{P_{t+1}^B}{P_t^B} + I_{t+1}^B \tag{25}$$

P_t^B is the advertising industry's output price index (PPI), available from the Bureau of Labor Statistics. Using the perpetual inventory method, we choose the initial stock according to:

$$U_0^B = \frac{I_0^B}{g^B + \delta^B}.$$

in which I_0^B is the firms' investment in brand capital in the first year in the sample. We choose g^B to match the average growth rate of advertising investments, which in our sample is about 10%. As in Vitorino (2014), we use a depreciation rate of 20%. Once we have the initial capital stock, we iterate forward using the depreciation rate, advertising investment amount, and investment price index. The investment rate on brand capital is then given by the ratio of the current period investment and the beginning of the period corresponding brand capital stock I_t^B/K_t^B.

Additional firm-level variables: Total debt, B_{it+1}, is long-term debt (item DLTT) plus short term debt (item DLC), setting them to zero where they are missing.. The market value of equity, P_{it}, is the closing price per share (item PRCC_F) times the number of common shares outstanding (item CSHO). For firms with different fiscal year ends the price matches the firm's fiscal year.

We measure the tax rate, τ_t, as the statutory corporate income tax (from the Commerce Clearing House, annual publications). Stock variables subscripted t ($t+1$ for debt) are measured and recorded at the end of year t, while flow variables subscripted t are measured over the course of year t and recorded at the end of year $t+1$.

Labor skill industry classification: Following Belo et al. (2017), we separate the economy into three broad industries based on the average labor skill of the workforce in the industry. Specifically, we classify an industry to be a low- medium, or high-skill industry based on the percentage of workers in that industry that work on occupations that require a high level of training and preparation (high-skill workers), using the Specific Vocational Preparation (SVP) index from the Dictionary of Occupational Titles (DOT), available from the Department of Labor, and employee data from the Bureau of Labor Statistics (BLS), Occupational Employment Statistics (OES) program. We use the data from Belo et al. (2017) to construct this industry classification. The base industry-level data is available at the three-digit Standard Industry Classification (SIC) level before and including year 2001, and at the four-digit North American Industry Classification System after 2001. An industry is classified as a high labor-skill industry if it belongs to a 3-SIC or 4-NAICS industry in which the percentage of high-skill workers in that industry (variable PSKILL) is above the 70$^{\text{th}}$ percentile of the cross-sectional distribution (across industries) of the PSKILL variable. We classify an industry as medium labor-skill industry if the percentage of high-skill workers in that industry is between the 30$^{\text{th}}$ and 70$^{\text{th}}$ percentile of the cross-sectional distribution of the PSKILL variable. Finally, we classify an industry as low labor-skill industry if the percentage of high-skill workers in that industry is below the 30$^{\text{th}}$ percentile of the cross-sectional distribution of the PSKILL variable.

Test assets: As noted, the estimation is performed at the portfolio-level. To guarantee that the estimates are not specific to a particular portfolio sort, we estimate the model combining a large number of test portfolios. The sorts are based on lagged values of all the ratios in the valuation ratio equation (21). Specifically, we consider four set of portfolios sorted on the lagged values of the following firm-level variables: the product of physical capital investment rate and scaled physical capital, $\left(\frac{I^P_{it-1}}{K^P_{it-1}}\right)\left(\frac{K^P_{it}}{A_{it-1}}\right)$, hiring rate time wages and the scaled labor force, $\left(\frac{H_{it-1}}{L_{it-1}}\right)\left(\frac{W_{it-1}L_{it}}{A_{it-1}}\right)$, the product of knowledge capital investment rate and scaled knowledge capital, $\left(\frac{I^K_{it-1}}{U^K_{it-1}}\right)\left(\frac{U^K_{it}}{A_{it-1}}\right)$, and the product of brand capital investment rate and scaled brand capital, $\left(\frac{I^B_{it-1}}{U^B_{it-1}}\right)\left(\frac{U^B_{it}}{A_{it-1}}\right)$. Sorting on these values is approximate equivalent to sort firms according to the (scaled) value of each capital/labor input, using a quadratic adjustment cost specification as an approximation.[13] We follow Fama and French (1993) in constructing the portfolios. Specifically, we sort all stocks in June of each year t into ten portfolios based on the deciles of the sorting variable of each firm for the fiscal year ending in $t-1$. The portfolios are rebalanced at the end of each June. This procedure gives a total of 40 portfolios across all sorts. In the robustness Section below, we also consider different number of portfolios on each sort (2, 5 and 20, thus a total of 8, 20 and 80 test assets, respectively).

The final sample includes data from 2,564 firms, for a total of 22,455 firm-year observations. The following data items restricted the sample size as follows (percentage numbers correspond to sequential dropping of the data). 17% of observations were dropped due to missing physical capital investment or physical capital stock data. Then, 2% of firm-year observations were dropped due to missing hiring rate and number of employees data. Then, 60% of the observations were dropped due to missing R&D expenses or knowledge capital stock data, and 0.2% of the observations were

[13]For example, according to equation 21 the value of physical capital is given by $q^P_{it}\frac{K^P_{it+1}}{A_{it+1}}$. Using equation (17) and a curvate parameter of 2 (quadratic), the previous value is:

$$q^P_{it}\frac{K^P_{it+1}}{A_{it+1}} = \left(1 + (1-\tau_t)\theta^2_P\left(\frac{I^P_{it}}{K^P_{it}}\right)\right)\frac{K^P_{it+1}}{A_{it+1}},$$

and hence our procedure captures the variation of the value of the physical capital stock across firms captured by the second term in the first brackets.

dropped due to missing advertising expenses or brand capital data (these two last numbers are relatively small because the firms that report advertise expenditures are usually a subset of the firms that report R&D expenses). Also 0.6% of observations were dropped due to missing equity price or bond data. Finally, 7.8% of the observations were dropped due to missing portfolio-sorting variable. The large number of observations dropped due to missing R&D (and advertising expenditures) data is expected given that some firms do not report separately R&D or advertising expenses from SG&A data, and is thus a well known problem with using these data items from Compustat. As noted in Section 3.2, the estimation approach at the portfolio-level, not firm-level, mitigates some of the concerns with this large number of missing observations because the portfolio-level moments are likely to be more stable with respect to firm exit and entry or other accounting issues than firm-level moments, and hence the method is likely to be more robust to missing data.[14]

3.4 Summary Statistics

Panel A in Table 1 reports the summary statistics (time series average of the cross sectional median, denoted as median, and standard deviation) of the valuation ratios and its components according to equation (21), across all firms in the economy, and separately in the low labor-skill and high labor-skill industries (we omit the intermediate labor-skill industry to save space).

The median valuation ratio across all firms is 1.97. This valuation ratio is higher in high labor-skill industries than in low labor-skill industries, 2.10 versus 1.31, respectively. Investment in knowledge capital has the highest median rate (27%), while investment in labor, the gross hiring rate, has the lowest median (16%). The investment and hiring rates are all higher in the high labor-skill industry than in the low labor-skill industry. In terms of volatility, the physical capital investment rate and the hiring rate are the two most volatile investment rates, with a standard deviation of 25% per annum.

[14]In unreported results (available upon request), we have constructed imputed measures of R&D and advertising expenses using SG&A data and other firm-level characteristics and industry fixed-effects for the firms with missing R&D and advertising expenses data. The results using these imputed measures, and hence using a significantly larger sample size, are overall consistent with the main results reported here. To be conservative, we do not use any imputed measures in the baseline empirical results reported here.

[Insert Table 1 here]

In terms of the average size of the scaled capital inputs, when computed across all firms, physical capital is the largest capital stock, with 44% of total assets (assets measured as the sum of the physical capital, knowledge capital, and brand capital inputs). The ratio of the wage bill (using lagged wages as implied by (21)) to total assets is 28%. The ratio of brand capital stock to total assets is 36%. The smallest scaled capital stock is knowledge capital, 11%. The relative magnitude of the ratios varies across the labor skill industries. For example, the scaled physical capital stock is higher in low labor-skill than in high labor-skill industries, 64% versus 41% of total assets, respectively. Conversely, the scaled brand capital stock is lower in low labor-skill than in high labor-skill industries, 13% versus 42% of total assets, respectively. Clearly, brand capital is more important in high labor-skill than in low labor-skill industries.

The shadow prices of the labor and capital inputs in equations (17) to (20) are determined by the investment/hiring rates of each input. Thus, understanding the properties of the investment/hiring rates is useful for understanding the time-series properties of the value the inputs. Panel B in Table 1 reports the investment and hiring rate cross-correlations.

The summary statistics reported in Panel B of Table 1 shows that, as expected, the investment/hiring rates are all positively correlated. These correlations range from a minimum of 31% for the correlation between hiring and investment in knowledge capital, to 48% for the correlation between investment in physical capital and in knowledge capital.

4 Empirical Results

This section reports the main empirical findings. Section 4.1 provides a firm-value decomposition based on the book-value of the capital inputs to provide a baseline decomposition. Section 4.2 reports the model's estimation results across all firms in the economy, thus assuming an homogeneous technology. In addition, this section provides a comparison of the model fit relative to simplified versions of the model with fewer capital inputs, including the one capital input model.

Section 4.3 reports the estimation results separately across low, medium, and high labor-skill industries, thus allowing for cross sectional heterogeneity in the firm's technology across industries.

4.1 Firm-Value Decomposition Based on Book Values

Before performing a formal estimation of the model, we can use the scaled capital input moments reported in Table 1 to make a preliminary assessment of the relative importance of each input for firm value based on the book-value of the inputs. If adjustment costs are zero, the shadow prices of the capital and labor inputs in equations (17) to (20) are simply 1, 0, $(1-\tau_t)$, and $(1-\tau_t)$. As a result, the value of each input is equal to its book-value, and the importance of each input for firm value can be computed from equations (12) to (15) without having to perform any estimation.

[Insert Table 2 here]

Table 2 reports the firm's book-value decomposition across all firms in the economy, and in the low and high labor-skill industry. To obtain this decomposition, we evaluate equations (12) to (15) at the median value of the ratio of the (scaled) capital inputs, and using the average tax rate in our sample of 38.1%. Without labor adjustment costs, as discussed in the Introduction section, the value of installed labor force is zero. Across all firms, the most important input is physical capital, which represents about 60.2% of the firm book-value. The second most important input is brand capital which represents 30.5% of the firm's book-value, and the last one is knowledge capital which represents about 9.3% of the firm's book-value. These numbers vary significantly across low and high labor-skill industries. The importance of physical capital for the book-value of the firm is significantly higher in the low labor-skill than in the high labor-skill industry, 78.1% versus 56.0%, respectively. Conversely, the value of brand capital is significantly lower in low skill than in high skill industries, 9.8% versus 35.5%, respectively.

In the presence of adjustment costs, the shadow prices of each input vary over time, and hence the relative importance of each input for firm's market value will be different from this baseline case. Thus, the difference in the importance of each input for firm-value relative to this benchmark no adjustment costs case is informative about the importance of the adjustment costs in each input.

4.2 Estimation Across All Firms

We first estimate the model using pooled data from all firms in the economy, thus assuming a homogeneous adjustment cost technology across firms.

Parameter Estimates and Model Fit

Panel A in Table 3, reports the point estimates of the adjustment cost parameters. Column (1) reports the baseline results. To help establish the importance of each input for the results, the remaining columns in this table provide the estimation results from several constrained specifications of the model using different subset of the capital and labor inputs, including the standard one capital input model.

[Table 3 here]

The estimate of the slope adjustment cost parameters (θ_i) are $\theta_P = 1.94$ for physical capital, $\theta_L = 3.45$ for labor, $\theta_K = 1.90$ for knowledge capital, and $\theta_B = 3.27$ for brand capital capital. All the slope adjustment cost coefficients are statistically significant, which implies that we cannot reject the hypothesis that these inputs are subject to positive adjustment costs.

The estimate of the curvature adjustment cost parameters (ν_i) are $\nu_P = 2.50$ for physical capital, $\nu_L = 1.64$ for labor, $\nu_K = 1.99$ for knowledge capital, and $\nu_B = 2.09$ for brand capital. The evidence thus suggest that for labor, the adjustment cost function has less curvature than the standard quadratic adjustment cost specification that, for tractability, is often used in the investment literature. For capital, the estimates suggest that the adjustment cost function has slightly more curvature than the quadratic adjustment cost specification. This result is consistent with the findings in BXZ who estimate a curvature parameter for the physical capital adjustment cost function that is significantly higher than 2. Finally, the evidence implies that the firm's optimization problem has an interior solution because the point estimates of both the physical capital, labor, and the two intangible capital adjustment cost parameters mean that the adjustment cost function is increasing and convex in the investment/hiring rates.

Turning to the analysis of the model fit, Panel B in Table 3 provides four measures of fit. It reports: i) the cross sectional R^2 (denoted XS-R^2) of a scatter plot of the average portfolio-level valuation ratio against the average portfolio-level predicted (model-implied) valuation ratio; ii) the time series R^2 measure of the pooled portfolio-level data (which is the measure that is implicitly targeted in the estimation); iii) the mean absolute errors (m.a.e.), computed as the means of the absolute errors of the error term of each portfolios; and iv) the m.a.e. as a fraction of the average valuation ratio of each portfolio (thus providing a relative measure of the size of the error term in each portfolio).

According to the four metrics considered here, the model performs well. The time series R^2 is 71%. In addition, the cross sectional R^2 is quite high, 94%, even tough the model estimation does not target this moment. Thus, the model performs well both in the cross sectional and in the time series dimensions. In terms of average valuation ratio, the model mean absolute error is quite low, only about 19% of the mean portfolio-level valuation ratio.

Figure 1 provides a visual description of the good fit of the model both in the time-series and in the cross section. Panel A shows the time series plot of the cross-sectional average (across the 40 portfolios used as test assets) portfolio median valuation ratio observed in the data (realized VR) and predicted by the model (predicted VR). Panel B shows the scatter plot across portfolios of the time series average of the cross-sectional median valuation ratio observed in the data against the predicted by the model.

[Figure 1 here]

The estimation procedure uses four different sorting variables to create the portfolios. As discussed in Section 3.3, the sorting variables capture the ex ante cross-sectional variation in the value of each capital and labor input. To show that the model matches the variation in valuation ratios in each one of the four portfolio sorts, Figure 2 shows the time series plot of the cross-sectional average (across the 10 portfolios in each portfolio sort) portfolio-level median valuation ratio observed in the data (realized VR) and predicted by the model (predicted VR). The model fit across these four sorts appears to be similar.

[Figure 2 here]

Firm-Value Decomposition and Adjustment Costs

The parameter estimates allows us to estimate the contribution of each capital input for firm's market value. In addition, the estimates allow us to determine the magnitude of the adjustment costs of the labor and capital inputs. Naturally, these two analysis are related. In this section, we provide an economic interpretation of the parameter estimates in terms of its implications for firm value decomposition and implied magnitude of adjustment costs.

To obtain the implications of the parameter estimates for firm's market value, we use the estimates reported in column 1, Panel A in Table 3 to compute, for each firm and in each year, the value of $q_{it}^P \frac{K_{it+1}^P}{A_{it+1}}$, $q_{it}^L \frac{L_{it+1}}{A_{it+1}}$, $q_{it}^K \frac{K_{it+1}^K}{A_{it+1}}$, and $q_{it}^B \frac{K_{it+1}^B}{A_{it+1}}$, that is, the scaled value of each capital/labor input. We then compute the cross sectional median value of the previous values and substitute these values in equations (12) to (15) to compute, in each year, the fraction of the firm value attributed to each capital/labor input. We interpret this procedure as capturing the firm-level decomposition for the median firm in the economy.[15]

Panel B in Table 3 (column 1, firm value decomposition), reports the time-series average of the previous fraction of the firm value attributed to each capital/labor input. The four inputs are important determinants of firm value. In the baseline specification, physical capital accounts for 29.6% of firms' market value, the installed labor force accounts for 27.2%, knowledge capital

[15] Alternatively, one could compute the fraction of firm-value attributed to each capital/labor input for each firm in the economy, take the cross-sectional median of these values, and report the time series average of this median. This procedure does not work here because the sum of the cross sectional median weights does not add to one.

accounts for 6.1%, and brand capital accounts for the remaining 37.1%. This analysis reveal that physical capital accounts for less than 50% of the firm's total value on average. Clearly, in the modern economy, intangible capital and labor are important determinants of firm value.

What explains the relatively high importance of labor and intangible capital inputs, in addition to physical capital, for firm value? For the intangible capital inputs part of the value comes from the book-value of the capital stocks as noted in Table 2. That is, even without adjustment costs, the intangible capital inputs contribute in a non-trivial way to the firm's market value due to the size of the capital stocks. With adjustment costs, however, the relative importance of the inputs changes due to its effect on the shadow prices of the capital stocks. For labor inputs, its contribution to firm value is directly related to the size of the labor adjustment costs. To understand the firm value decomposition estimates, here, we evaluate the economic magnitude of the adjustment costs of the four inputs. Naturally, when an input is costly to adjust, installed values of the inputs are valuable to the firm because it contributes not only for production but also by allowing the firm to avoid adjustment costs in the future.

Panel B in Table 3 (column 1, adjustment costs), reports the implied proportion of firms' sales that is lost due to physical capital, labor, and intangible capital adjustment costs. Using the functional form specification in equation (16), these values are computed as a fraction of firm's total sales Y_{it} as follows:

$$\frac{CP_{it}}{Y_{it}} = \frac{\frac{1}{\nu_P}\left|\theta_P \frac{I^P_{it}}{K^P_{it}}\right|^{\nu_P} K^P_{it}}{Y_{it}} \tag{26}$$

$$\frac{CL_{it}}{Y_{it}} = \frac{\frac{1}{\nu_L}\left|\theta_L \frac{H_{it}}{N_{it}}\right|^{\nu_L} W_{it}L_{it}}{Y_{it}} \tag{27}$$

$$\frac{CK^K_{it}}{Y_{it}} = \frac{\frac{1}{\nu_U}\left|\theta_K \frac{I^K_{it}}{U^K_{it}}\right|^{\nu_K} U^K_{it}}{Y_{it}} \tag{28}$$

$$\frac{CK^B_{it}}{Y_{it}} = \frac{\frac{1}{\nu_B}\left|\theta_B \frac{I^B_{it}}{U^B_{it}}\right|^{\nu_B} U^B_{it}}{Y_{it}}. \tag{29}$$

We compute the adjustment costs estimates in an analogous way to the computation of the fractions of firm value. Specifically, we first compute the value in equations (26) to (29) for each firm and in each year. Then, in each year, we compute the cross sectional median of the previous values, and report the time-series average of these medians.

The estimated magnitude of labor and, to a less extent, knowledge capital adjustment costs is large, whereas the magnitudes of physical capital and brand capital adjustment costs is more modest. On average, the fraction of (annual) sales that is lost due to labor adjustment costs is 8.9%. The fraction of sales that is lost due to knowledge capital adjustment costs is 3.1%, and for brand capital is 1.8%. The fraction of sales that is lost due to physical capital adjustment costs is estimated to be low, 0.6%. Although there is no consensus on the magnitude of labor and capital adjustment costs, the estimated values of adjustment costs for these two inputs are within the empirical estimates surveyed in Hamermesh and Pfann (1996), and discussed in Merz and Yashiv (2007). For brand capital, the estimated value of adjustment costs is lower than those estimated in Vitorino (2014) (on average, about 8% of firm's annual sales). The fundamental difference is that we are estimating firm-level parameters whereas Vitorino (2014) estimates portfolio-level parameters.

Taken together, these point estimates show that labor is the input that is subject to the highest adjustment costs, which explains why the value of installed labor is an important components of the firms' market value. Because of the size of labor adjustment costs, the firm value decomposition based on the real shadow prices of the capital and labor inputs (and hence of the market value of the inputs) differs significantly from the firm value decomposition based on book-value of the inputs reported in Table 2.

Model Comparison

To help understand the fit of the model and the relative importance of each capital input for firm's valuation, we estimate restricted versions of the model using different subsets of the inputs. Table 3, column (2), reports the estimation results for the one-capital (physical capital) input model, which is the model used in BXZ and in LWZ. Column (3), reports the estimation results for a version of the model with only physical capital and labor inputs. Column (4) reports the estimation results for a version of the model with only physical capital and knowledge capital inputs. Finally, column (5) reports the estimation results for a version of the model with only physical capital and brand capital inputs. To provide a meaningful comparison of the model fit in terms of R^2, we use the same set of firms in the estimation of all models (that is, the sample is the same as the sample used for the estimation of the baseline model), and the observed valuation ratio of each firm (the variable we want to explain) is the same across models, that is, it is scaled by the same sum of the capital inputs (A_t) (so that the variation in the variable we want to explain is the same across models).

Panel A in Table 3, columns (2) to (5), shows that the estimate of the slope adjustment cost parameters (θ_i) and curvature parameters (ν_i) across the different model specifications are reasonable and do not differ significantly from the estimates in the baseline model. Interestingly, in terms of model fit, Panel B in Table 3, column (2) shows that the standard one-physical capital input model does a reasonable job explaining the cross-sectional variation in the average valuation ratio across the portfolios with a high cross-sectional R^2 of 64%. Although smaller than in the baseline model (94%), this high cross-sectional R^2 is consistent with the evidence in BXZ that the one-capital input model can capture well the cross-sectional variation in the valuation ratios in the economy. But the one-capital input model fails in explaining the time-series variation in the variation ratios. The time-series R^2 of the one-capital input model is only 9%, versus 71% in the baseline model. Figure 3 provides a visual description of the model fit of the one capital input model. Clearly, this model is unable to capture the time-series variation of firm's market value, although is able to capture some of the cross-sectional variation in the valuation ratios across portfolios. Thus, we conclude that the benefit of incorporating additional quasi-fixed inputs in the neoclassical investment model comes primarily from improving the model's ability to capture the time-series variation in the valuation ratios.

[Figure 3 here]

Turning to the analysis of the other model specifications, Panel B in Table 3, column (3), shows that adding labor to the one-capital input model has a first order impact on the model's fit. Comparing columns (2) and (3), the cross-sectional R^2 increases slightly from 64% to 76%, but the time-series R^2 increases substantially from 9% to 53% when quasi-fixed labor is added to the one-capital input model. Confirming the importance of labor for the good model' fit, Panel B, column (3) shows that in this model, the average fraction of firm value attributed to labor is quite high, about 56.6%, which is higher than the fraction of firm value attributed to physical capital. Adding brand capital also has an important impact on the model's fit, although slightly smaller than the impact of labor. Comparing columns (2) and (5), the cross-sectional R^2 does not change, but the time-series R^2 increases from 9% to 46% when brand capital is added to the one-capital input model. Confirming the importance of brand capital for the good model' fit, Panel B, column (5) shows that the average fraction of firm value attributed to brand capital is quite high, about 53.1%. The improvement in the model's fit due to the addition of knowledge capital is more modest. Comparing columns (2) and (4), the cross-sectional R^2 increases from 64% to 72%, and the time-series R^2 increases slightly from 9% to 14% when knowledge capital is added to the one-capital input model.

4.3 Estimation Across Industries

In the Section we evaluate the relative importance of the capital and labor inputs for firm's market value across industries.

We perform the estimation separately across low- medium- and high-skill industries, thus allowing for heterogeneity in the adjustment cost technology across firms (see Section 3.3 for the details on the construction of this industry classification). This industry classification (relative to other industry classifications available in the literature) is interesting for the purposes of our analysis because there is a priori reasons to expect that the adjustment cost parameters, and hence the importance of each capital and labor inputs for firm value, to vary in a systematic way across these industries. First, as discussed in Belo et al. (2017) (and see references therein) previous empirical studies find that it is more costly to replace a high-skill worker than a low-skill worker. Thus, this suggests that the labor adjustment cost parameters should differ across these industries, in particular, they should be higher in high labor-skill industries. All else equal, the higher labor adjustment cost parameter implies that labor should represent a higher fraction of firm value in high labor-skill industries. Second, Belo et al. (2017) provide evidence that investment in intangible capital inputs such as R&D expenditures is higher in high labor-skill than in low labor-skill industries. Taken together, this suggests that the relative importance of the different capital and labor inputs for firm value should vary across these industries.

Parameter Estimates and Model Fit

Panel A in Table 4, columns (1) to (3), report the point estimates of the adjustment cost parameters in the low (L), medium (M), and high (H), labor-skill industry, respectively. To help establish the importance of each capital/labor input for in each industry, the remaining columns in this table provide the estimation results from several alternative specifications of the model, in particular, restricted versions of the model with different subsets of the capital and labor inputs.

[Table 4 here]

The estimate of the slope adjustment cost parameter for labor increases with the average labor-skill level of the industry, from $\theta_L = 1.70$ in the low skill industry to $\theta_L = 3.38$ in the high skill industry. Going in the opposite direction, the slope adjustment cost parameters for physical capital, knowledge capital, and brand capital decrease the average labor-skill level of the industry. Turning to the analysis of the curvature adjustment cost parameters, the curvature adjustment cost parameter for physical capital and brand capital increase with the average labor-skill level of the industry. The curvature adjustment cost parameters for labor and knowledge capital do not exhibit a systematic pattern across these industries.

Turning to the analysis of the model fit, Panel B in Table 4, columns (1) to (3), report four metrics of the model fit in each labor-skill industry. The model performs particularly well in explaining the cross-sectional and time-series variation in the high-skill industry, with a cross sectional R^2 of 94%, and a time-series R^2 of 67%. In terms of average valuation ratio, the model mean absolute error in the high labor-skill industry is only 20% of the mean valuation ratio in that industry. The model fit in low skill industry is more modest but still reasonable, with a cross sectional R^2 of 74%, and a time-series R^2 of 38%. In terms of average valuation ratio, the model mean absolute error in the low labor-skill industry is about 37% of the mean valuation ratio in the low-skill industry.

Comparing across model specifications, Panel B in Table 4 confirms the conclusions from the estimation of the model across all firms: adding labor to the one-capital input model has a first order impact on the model's fit. The new finding here is that this improvement is significantly more pronounced in the high labor-skill industry. For tractability, we focus our discussion here on the time-series R^2, because its the most informative for this analysis due to its substantial variation across model specifications (also, for the alternative model specifications, we only report the estimation results in the low and high labor-skill industries, and omit the results for the medium labor-skill industry to save space). Comparing columns (5 and 7, the time-series R^2 in the high labor-skill industry increases from 7% to 52% when quasi-fixed labor is added to the one-capital input model. Comparing columns (4) and (6), the time-series R^2 in the low labor-skill industry increases slightly from 2% to 10% when quasi-fixed labor is added to the one-capital input model. Similarly, adding brand capital also has an important impact on the model's fit, and this improvement is also concentrated in the high labor-skill industry. Comparing columns (5) and (11), the time-series R^2 increases from 7% to 41% when brand capital is added to the one-capital input model. Comparing columns (4) and (10), the time-series R^2 increases very slightly from 2% to 3% when brand capital is added to the one-capital input model. Finally, different from the previous analyses, the improvement from adding knowledge capital to the one-capital input model is more concentrated in the low labor-skill industry. Comparing columns (5) and (9), the time-series R^2 increases very slightly from 7% to 13% when knowledge capital is added to the one-capital input model. Comparing columns (4) and (8), the time-series R^2 increases from 2% to 26% when brand capital is added to the one-capital input model.

Figure 4 provides a visual description of the fit of the model in the low and high labor-skill industry, both in the time-series and in the cross section. A comparison of Panels B and D shows that the variation in the average valuation ratios across portfolios is significantly higher in the high labor-skill than in the low labor-skill industry. Panel D shows that the model can match this large variation quite well.

[Figure 4 here]

Firm-Value Decomposition and Adjustment Costs

To provide an economic interpretation of the previous estimates, we investigate its implications for the variation in the firm value decomposition and adjustment cost estimates across industries.

Confirming the importance of labor for the good model' fit, especially in the high labor-skill industry, Panel B in Table 4, columns (1) to (3) shows that the average fraction of firm value attributed to labor increases with the average labor-skill level of the industry. In the low labor-skill industry, the fraction of firm value that can be attributed to labor is on average only 6.6%, whereas in the high-skill industry this fraction is 29.9%. Similarly, the fraction of firm value attributed to brand capital also increases with the average labor-skill level of the industry (despite the decrease in the brand capital adjustment cost slope coefficient reported in Panel A). In the low labor-skill industry, the fraction of firm value that can be attributed to brand capital is on average only 14.1%, whereas in the high labor-skill industry this fraction is 39.9%. Finally, going in the opposite direction, the fraction of firm value attributed to knowledge capital decreases with the average labor-skill level of the industry. In the low labor-skill industry, the fraction of firm value that can be attributed to knowledge capital is on average 18.8%, whereas in the high labor-skill industry this fraction drops to 5.3%.

Turning the analysis to the variation in the size of adjustment costs across industries, Panel B in Table 4, columns (1) to (3) shows that the estimated labor adjustment costs increase significantly with the average labor-skill level of the industry. The fraction of (annul) sales lost due to labor adjustment costs are on average 11.5% in the high labor-skill industry, and only 1.6% in the low labor-skill industry. Similarly, the estimated knowledge capital adjustment costs increase (although significantly less than for labor) with the average labor-skill level of the industry. The fraction of sales lost due to knowledge capital adjustment costs are on average 3.5% in the high labor-skill industry, and only 1% in the low labor-skill industry. This positive relationship between size of adjustment costs and average labor-skill of the industry is reversed for physical capital and brand capital inputs. The fraction of sales lost due to physical capital adjustment costs are on average 0.5% in the high labor-skill industry, and 5.3% in the low labor-skill industry. Similarly, the fraction of sales lost due to brand capital adjustment costs are on average 1.2% in the high labor-skill industry, and 7.6% in the low labor-skill industry.

Taken together, this analysis shows that the relative importance of the capital/labor inputs exhibits substantial variation across the labor-skill industries, and allowing for technology heterogeneity across industries seems important for a proper characterization of the importance of each capital and labor inputs for firm value at a more disaggregated industry level. In addition, the results show that adding additional inputs to the baseline one-capital input model is especially important in high labor-skill industries. While in low labor-skill industries, the value of physical capital represents about 60% of firm's market value, in high labor-skill industries the value of physical capital represents less than 25% of the firm's value. Thus, in high labor-skill industries the majority of the firm's market value (about 75%) can be attributed to the other inputs, namely, brand capital, labor, and, to a less extent, knowledge capital.

5 Time-Series and Risk Characteristics of Labor and Capital Inputs

In this section we use the parameter estimates obtained in the previous section to perform additional analysis. First, we evaluate if the importance of each capital/labor input for firm value has changed over time. Second, we evaluate the business cycle properties of the value of each capital/inputs. This analysis is useful because it allow us to understand the risk properties of each capital/labor input, and hence of firm's market value.

5.1 Value Decomposition Across Decades

The analysis in the previous Section reports the time-series averages of the firm value decomposition in the full sample from 1975 to 2013. To provide a more detailed characterization of the data, here we perform the same analysis across sub-periods (we do not re-estimate the parameters value because the model assumes they are constant over time). We perform the analysis using the estimates obtained using all firms in the economy or separately estimated across labor-skill industries. To compute the fraction of firm value attributed to each capital input across all firms we use the estimates from column (1), Panel A in Table 3, and across labor-skill industries, we use the estimates from columns (1) to (3), Panel A, in Table 4.

[Table 5 here]

Table 5 reports the time series averages the fraction of firm value attributed to each capital inputs across decades: 70s (1975-1979), 80s (1980-1989), 90s (1990-1999), 00s (2000-2009), and 10s (2010-2013). Figure 5 provides a visual description of the trends in the input value shares in the data, both across all firms, and in the low and high labor-skill industries.

[Figure 5 here]

Across all firms, the table and the figure allows us to identify two strong patterns in the data. First, the importance of physical capital input has decreased over our sample period from 48.6% in the 70s to 18.2% in the 10s. Second, in the opposite direction, the importance of labor input and brand capital for firm value has increased over our sample period. The contribution of labor for firm value increased from 20.4% in the 70s, to 36.4 in the 10s. The contribution of brand capital for firm value increased from 20.4% in the 70s, to 36.4% in the 10s. The contribution of knowledge capital for firm value was relatively small during the entire period, although is has slightly decreased over the sample period.

Turning to the analysis of the change in the importance of each input for firm's market value across labor skill industries, Table 5 allows us to identify interesting differences across industries. Even tough the increase in the importance of labor and brand capital, and the corresponding decrease of physical capital for firm value is pervasive across all industries, the trends are significantly more pronounced in the mid and, specially, in the high labor-skill industries. Also, the slight decrease in the importance of knowledge capital for firm value is concentrated in the mid and high-labor skill industries, only. In the low labor-skill industry, the importance of knowledge capital for firm value has actually increases.

Taken together, the analysis in this section highlights the importance of labor and brand capital for understanding firm value, especially in the recent decades, and in high labor-skill industries. The increase in the importance of labor for firm value resembles the evidence in Hartman-Glaser, Lustig, and Xiaolan (2017) who show that the cross sectional average labor share of publicly traded firms has increased over time in the U.S. economy (in contrast with the well documented decrease of the *aggregate* labor share over the same sample period, as noted in Elsby, Hobijn, and Şahin 2013, Karabarbounis and Neiman 2013, among others). The difference is that we compute the importance of the value of labor for firm-value, not for value added as in Hartman-Glaser, Lustig, and Xiaolan (2017). Finally, the compositional change in the importance of each input for firm value highlights the importance of targeting the time series of the valuation ratios in the estimation, as opposed to only targeting the cross sectional time series means of the valuation ratios, as in LWZ/BXZ.

5.2 Risk Characteristics of Labor and Capital Inputs

In addition to the analysis of the contribution of each input for firm value, the parameter estimates allows us to characterize the business-cycle properties of the value, and corresponding firm-value shares, of each input. This analysis is useful because it allows us to understand the risk characteristics of the inputs, and hence the risk characteristics of the firm.

We proceed as follows. Given the parameters estimates, we compute for each firm the time series of the firm's model-implied valuation ratio, as well as the time series of the (scaled) value of each capital input. These series are given by:

$$
\begin{aligned}
V_{it}^{P} &= \hat{q}_{it}^{P}\frac{K_{it+1}^{P}}{A_t} : \text{ value of physical capital} \\
V_{it}^{L} &= \hat{q}_{it}^{L}\frac{L_{it+1}}{A_t} : \text{ value of labor} \\
V_{it}^{K} &= \hat{q}_{it}^{K}\frac{U_{it+1}^{K}}{A_t} : \text{ value of knowledge capital} \\
V_{it}^{B} &= \hat{q}_{it}^{B}\frac{U_{it+1}^{B}}{A_t} : \text{ value of brand capital} \\
VR_{it} &= V_{it}^{P} + V_{it}^{L} + V_{it}^{K} + V_{it}^{B}
\end{aligned}
$$

Then, in each year, and consistent with the approach in the previous Sections, we compute the cross-sectional median of each component, and also of the firm's valuation ratio (VR). We then use these values to compute the share of each input for firm value. Because we are interested in understanding the business cycle properties of these components, we then extract the cyclical component of the log of the previous variables through an HP filter (with smoothing factor of 100). The cyclical components are measured in percentage deviation relative to the trend. We also extract the cyclical component of aggregate sales using a similar procedure. To understand the volatility and the cyclicality of the input shares and values, we then compute the correlation of the cycle component of each input share and value with the business cycle, measured by the cycle in aggregate sales ($\mathrm{Y}^{\mathrm{agg}}$). We compute the moments across all firms in the economy, and also separately in low and high labor-skill industries.

[Table 6 here]

Table 6 reports the results from this analysis. Panel A shows that, as expected, the valuation ratio is procyclical (correlation with aggregate sales is 41%), especially in the high labor-skill industry. The analysis of the correlation of the cyclical components of capital and labor firm-value shares with aggregate sales across all firms reveals an interesting pattern. While the share of labor on firm value is procyclical (positive correlation), the shares of the capital inputs is countercyclical. The analysis across industries shows that this pattern is mostly driven by the firms in the high labor-skill industry (in the low skill industry, only the brand capital share is countercyclical). Thus, the importance of the labor input for understanding firm's market value is higher during good economic times.

Turning to the analysis of cyclical components of the scaled value of capital/labor inputs, Table 6 also shows that across all firms, all input values are procyclical (across industries, brand capital value is countercyclical in low skill industries and physical capital is slightly countercyclical in high skill industries.), especially labor and knowledge capital inputs. This result means that the value of labor and knowledge capital are the ones most affected by recessions, but that benefit the most from expansions. Thus, the value of the labor input and knowledge capital seem to be the most sensitive to aggregate economic conditions.

The previous result is consistent with the cyclicality of the capital/labor input shares in firm's value. In good times, all values good up, but the value of labor increases by more than the value of physical and brand capital (knowledge capital is less important because it has a smaller weight in the value decomposition). As a result, the share of labor goes up, but the share of physical and brand capital goes down. These patterns are reinforced by the higher volatility of the cyclical component of labor (share and value). Across all firms, the standard deviation of the cyclical component of the share of labor inputs is 0.25, which is at least 2 times more volatile than the cyclical component of the share of the other inputs (all below 0.10 across all firms). Similarly, the standard deviation of the cyclical component of the scaled value of labor inputs is 0.31, which is at least 4 times more volatile than the cyclical component of the share of the scaled value of the other inputs (all below 0.07 across all firms).

Taken together, our analysis shows that the value of labor is the most volatile component of firm value, and it is also, together with knowledge capital, the most cyclical component. Thus, understanding the dynamics of labor inputs across firms seems important for understanding the dynamics of firm value.

6 Robustness

To check the robustness of the main findings, we re-estimate the parameters of the model across several perturbations of the empirical procedures. Specifically, we re estimate the model parameters using a different number of portfolios as test assets, and using other portfolio-level moments, not just the cross sectional median. We also report the results from firm-level estimation. To facilitate the analysis, and avoid a proliferation of tables, we estimate the model parameters using all firms in the economy.

6.1 Different Number of Test Assets

Panel A in Table 7, columns (2) to (4), reports the estimation results using a different number of portfolios as test assets. In the baseline estimation, we use 10 portfolios for each one of the 4 portfolio sorts (so a total of 40 portfolios). In column (2) we consider 2 portfolios for each portfolio sort (a total of 8 portfolios), in column (3) we consider 5 portfolios for each portfolio sort (a total of 20 portfolios), and in column (4)we consider 20 portfolios for each portfolio sort (a total of 80 portfolios).

[Insert Table 7 here]

The point estimates reported in columns (2) to (4) appear to be similar in magnitude to the point estimates in the baseline case, reported in column (1). But its difficult to judge the degree of similarity of the estimates based on these point estimates only. To help the interpretation of the results, its useful to focus our analysis on the differences between the fractions of firm value implied by each set of point estimates. As reported in Panel B in Table 7, the contribution of each input for firm value is relatively stable across columns. This analysis suggest that the point estimates in the baseline case are robust to reasonable variation of the number of portfolios used in the estimation.

6.2 Different Target Moments

The baseline estimation of the model matches the behavior of the median firm in each portfolio. As discussed in Section 3.2, we also estimate the model by targeting a different set of cross sectional moments. In column (5) we estimate the model parameters by targeting the portfolio-level cross sectional equal-weighted average (XSEW), instead of the cross sectional median (XSMED) used in the baseline estimation. In addition to this method, in columns (6) and (7) we target alternative cross-sectional moments. In column (6), the estimation targets the portfolio-level inter-quartile valuation ratio spread (VR_{75-25}), and in column (7), the estimation targets not only the cross sectional median (VR_{50}) but also the 25^{th} and 75^{th} percentiles of the portfolio-level cross sectional distribution of valuation ratios. Finally, in column (8) we drop the portfolio-level approach completely, and estimate the model parameters using firm-level data.

One technical issue arises in the estimation of the model at the firm-level or when we target the portfolio-level cross sectional average of firms' valuation ratios. These estimation approaches are very sensitive to outliers in the data, in contrast with the baseline estimation approach which targets the cross sectional median. So, in the results reported in columns (5) and (8), we use data winsorized at the top and bottom (if the variable admits negative values) 2% of the distribution of all the ratios included in the estimation. Recall that in the baseline estimation the data is not winsorized. That is one reason why we adopt the cross-sectional median estimation method as the primary estimation method.

The point estimates reported in Panel A of Table 7, columns (5) to (8), appear to be similar in magnitude to the point estimates in the baseline case, reported in column (1). To help the interpretation of the results, it is useful to focus our analysis on the differences between the fractions of firm value implied by each set of point estimates. As reported in Panel B in Table 7, the contribution of each input for firm value is stable across columns (5) to (7). In column (8), using the firm-level estimation, the fraction of firm value attributed to labor is somewhat lower than in the baseline case: 14.5% here versus 27.2% in the baseline case. This result is expected if there is substantial measurement error in firm-level labor data: in this case, we expect the adjustment cost parameter estimates to be biased towards zero, and hence the firm-value decomposition is closer to the firm book-value decomposition discussed in Section 4.1.

Taken together, the previous analyses show that main empirical results are robust to reasonable variations of the empirical procedures.

7 Conclusion

We incorporate quasi-fixed labor, knowledge capital and brand capital, into the neoclassical model of investment, and estimate the contribution of each input for explaining firms' market values. The structural model performs well in explaining both cross-sectional and time-series variation of firms' market value, with a time series R^2 of 71% and a cross sectional R^2 of 94%. Across all firms, physical capital accounts for 29.6% of firms' value, installed labor force accounts for 27.2%, knowledge capital accounts for 6.1%, and brand capital for 37.1%. We show that financial markets assign large and positive values to the installed stocks of the different types of inputs because they are costly to adjust, allowing firms to extract some rents to compensate firms for the cost of adjusting the inputs. Overall, our value decomposition provides direct empirical evidence supporting models with multiple capital inputs as main sources of firm value.

Our estimation results allow us to characterize the time-series and business cycle of properties of the market value of the different capital inputs. We document that the importance of physical capital has decreased substantially over the decades, while the importance of labor and brand capital input has increased. Furthermore, the value of labor is more volatile and procyclical than the value of the remaining inputs, which suggest that understanding the dynamics of firms's labor inputs is useful for understanding the dynamics of firm values.

Finally, methodologically, our estimation procedure targets portfolio-level cross-sectional moments allows us to estimate firm-level structural parameters and avoid the aggregation bias of the BXZ and LWZ estimation procedure. This is useful for practical applications because it allows us to compute market values at the firm- not portfolio-level, which is naturally more useful in practice. Possible applications include the valuation of private firms or initial public offerings, provide guidance in merger and acquisition transactions, among other applications that require estimates of firm values.

Appendix

A Derivation: Firm Value Decomposition

The first order conditions with respect to I_{it}^P, K_{it+1}^P, H_{it}, L_{it+1}, I_{it}^K, U_{it+1}^K, I_{it}^B, U_{it+1}^B, and B_{it+1}, from maximizing the cum-dividend market value of equity are:

$$q_{it}^P = 1 + (1-\tau_t)\frac{\partial C_{it}}{\partial I_{it}^P} \tag{A.1}$$

$$q_{it}^P = E_t\left[M_{t+1}\left[(1-\tau_{t+1})(\frac{\partial \Pi_{it+1}}{\partial K_{it+1}^P} - \frac{\partial C_{it+1}}{\partial K_{it+1}^P}) + \delta_{it+1}^P\tau_{t+1} + (1-\delta_{it+1}^P)q_{it+1}^P\right]\right] \tag{A.2}$$

$$q_{it}^L = (1-\tau_t)\frac{\partial C_{it}}{\partial H_{it}} \tag{A.3}$$

$$q_{it}^L = E_t\left[M_{t+1}\left[(1-\tau_{t+1})(\frac{\partial \Pi_{it+1}}{\partial L_{it+1}} - \frac{\partial C_{it+1}}{\partial L_{it+1}} - W_{it+1}) + (1-\delta_{it+1}^L)q_{it+1}^L\right]\right] \tag{A.4}$$

$$q_{it}^K = (1-\tau_t)\left[1 + \frac{\partial C_{it}}{\partial I_{it}^K}\right] \tag{A.5}$$

$$q_{it}^K = E_t\left[M_{t+1}\left[(1-\tau_{t+1})(\frac{\partial \Pi_{it+1}}{\partial U_{it+1}^K} - \frac{\partial C_{it+1}}{\partial U_{it+1}^K}) + (1-\delta_{it+1}^K)q_{it+1}^K\right]\right] \tag{A.6}$$

$$q_{it}^B = (1-\tau_t)\left[1 + \frac{\partial C_{it}}{\partial I_{it}^B}\right] \tag{A.7}$$

$$q_{it}^B = E_t\left[M_{t+1}\left[(1-\tau_{t+1})(\frac{\partial \Pi_{it+1}}{\partial U_{it+1}^B} - \frac{\partial C_{it+1}}{\partial U_{it+1}^B}) + (1-\delta_{it+1}^B)q_{it+1}^B\right]\right] \tag{A.8}$$

$$1 = E_t\left[M_{t+1}\left[r_{it+1}^B - (r_{it+1}^B - 1)\tau_{t+1}\right]\right] = E_t\left[M_{t+1}r_{it+1}^{Ba}\right] \tag{A.9}$$

In the last equation we defined the after-tax bond return as $r_{it+1}^{Ba} \equiv r_{it+1}^B - (r_{it+1}^B - 1)\tau_{t+1}$.

Using the FOCs (A.2, A.4, A.6, and A8),

$$\begin{aligned}
& q_{it}^P K_{it+1}^P + q_{it}^L L_{it+1} + q_{it}^K U_{it+1}^K + q_{it}^B U_{it+1}^B \\
= & E_t\left[M_{t+1}\left[(1-\tau_{t+1})(\frac{\partial \Pi_{it+1}}{\partial K_{it+1}^P}K_{it+1}^P + \frac{\partial \Pi_{it+1}}{\partial L_{it+1}}L_{it+1} + \frac{\partial \Pi_{it+1}}{\partial U_{it+1}^K}U_{it+1}^K + \frac{\partial \Pi_{it+1}}{\partial U_{it+1}^B}U_{it+1}^B)\right.\right. \\
& - (1-\tau_{t+1})(\frac{\partial C_{it+1}}{\partial K_{it+1}^P}K_{it+1}^P + \frac{\partial C_{it+1}}{\partial L_{it+1}}L_{it+1} + \frac{\partial C_{it+1}}{\partial U_{it+1}^K}U_{it+1}^K + \frac{\partial C_{it+1}}{\partial U_{it+1}^B}U_{it+1}^B) \\
& + (1-\delta_{it+1}^P)q_{it+1}^P K_{it+1}^P + (1-\delta_{it+1}^L)q_{it+1}^L L_{it+1} + (1-\delta_{it+1}^K)q_{it+1}^K U_{it+1}^K + (1-\delta_{it+1}^B)q_{it+1}^B U_{it+1}^B \\
& \left.\left. + \delta_{it+1}^P\tau_{t+1}K_{it+1}^P - (1-\tau_{t+1})W_{it+1}L_{it+1}\right]\right]
\end{aligned}$$

With constant return to scale production and adjustment costs,

$$
\begin{aligned}
& q_{it}^P K_{it+1}^P + q_{it}^L L_{it+1} + q_{it}^K U_{it+1}^K + q_{it}^B U_{it+1}^B \\
= & E_t \left[M_{t+1} \left[(1-\tau_{t+1})(\Pi_{it+1} - C_{it+1} - I_{it+1}^K - I_{it+1}^B - W_{it+1} N_{it+1}) - I_{it+1}^P + \delta_{it+1}^P \tau_{t+1} K_{it+1}^P \right.\right. \\
& + (1-\tau_{t+1}) \frac{\partial C_{it+1}}{\partial I_{it+1}^P} I_{it+1}^P + I_{it+1}^P + (1-\tau_{t+1}) \frac{\partial C_{it+1}}{\partial H_{it+1}} H_{it+1} + (1-\tau_{t+1}) \frac{\partial C_{it+1}}{\partial I_{it+1}^K} I_{it+1}^K \\
& + I_{it+1}^K + (1-\tau_{t+1}) \frac{\partial C_{it+1}}{\partial I_{it+1}^B} I_{it+1}^B + I_{it+1}^B \qquad \text{(A.10)} \\
& + (1-\delta_{it+1}^P) q_{it+1}^P K_{it+1}^P + (1-\delta_{it+1}^L) q_{it+1}^L L_{it+1} + (1-\delta_{it+1}^K) q_{it+1}^K U_{it+1}^K \\
& \left.\left. + (1-\delta_{it+1}^B) q_{it+1}^B U_{it+1}^B \ \right]\right] \qquad \text{(A.11)} \\
= & E_t \left[M_{t+1} \left[(1-\tau_{t+1})(\Pi_{it+1} - C_{it+1} - I_{it+1}^K - I_{it+1}^B - W_{it+1} N_{it+1}) - I_{it+1}^P + \delta_{it+1}^P \tau_{t+1} K_{it+1}^P + B_{it+2} - r_{it+1}^B B_{it+1} \right.\right. \\
& \left.\left. + q_{it+1}^P K_{it+2}^P + q_{it+1}^L L_{it+2} + q_{it+1}^K U_{it+2}^K + q_{it+1}^B U_{it+2}^B - B_{it+2} \ \right]\right] + E_t \left[M_{t+1} r_{it+1}^{Ba} \right] B_{it+1}
\end{aligned}
$$

Rearranging the above equation,

$$
q_{it}^P K_{it+1}^P + q_{it}^L L_{it+1} + q_{it}^K U_{it+1}^K + q_{it}^B U_{it+1}^B - B_{it+1} = E_t \left[M_{t+1} \left[\begin{array}{c} D_{it+1} + q_{it+1}^P K_{it+2}^P \\ + q_{it+1}^L L_{it+2} + q_{it+1}^K U_{it+2}^K + q_{it+1}^B U_{it+2}^B - B_{it+2} \end{array} \right] \right]
$$

Recursively applying the above the equation to future periods,

$$
\begin{aligned}
& q_{it}^P K_{it+1}^P + q_{it}^L L_{it+1} + q_{it}^K U_{it+1}^K + q_{it}^B U_{it+1}^B - B_{it+1} \\
= & E_t \left[M_{t+1} D_{it+1} + M_{t+2} D_{it+2} + M_{t+2} \left[q_{it+2}^P K_{it+3}^P + q_{it+2}^L L_{it+3} + q_{it+2}^K U_{it+3}^K + q_{it+2}^B U_{it+3}^B - B_{it+3} \right] \right] \\
= & \ldots \\
= & \sum_{\triangle t=1}^{\infty} M_{t+\triangle t} D_{it+\triangle t} + \lim_{\triangle t \to \infty} E_t \left[M_{t+1} \left[q_{it+\triangle t}^P K_{it+\triangle t}^P + q_{it+\triangle t}^L L_{it+\triangle t} + q_{it+\triangle t}^K U_{it+\triangle t}^K + q_{it+\triangle t}^B U_{it+\triangle t}^B - B_{it+\triangle t} \right] \right]
\end{aligned}
$$

Assuming the transversality condition holds,

$$
q_{it}^P K_{it+1}^P + q_{it}^L L_{it+1} + q_{it}^K U_{it+1}^K + q_{it}^B U_{it+1}^B = V_{it} - D_{it} + B_{it+1} = P_{it} + B_{it+1}
$$

B Aggregation Bias in BXZ/LWZ and Alternative Estimation Procedures

In this Appendix, we use artificial data to investigate the ability of the different estimation approaches to recover the underlying firm-level structural parameters. We document that the parameter estimates using the aggregation procedure in Liu, Whited and Zhang (2009) (LWZ) do not have a structural interpretation. In addition, we verify that the alternative portfolio-level estimation methods proposed in the main text allows us to recover the firm-level structural parameters.

For simplicity, we consider the one-capital input model. To proceed, we generate data from a model economy in which the assumptions of the baseline investment model hold (and hence the firm-level observed and predicted (model-implied) valuation ratios are equal). But instead of simulating data from a model economy, we use the real data as follows. We construct the capital stock process for each firm by using the law of motion:

$$K_{it} = (1-\delta)K_{it-1} + I_{it}. \tag{B.1}$$

We use the firm-level physical capital investment data for I_{it} and the initial capital stock of the firm to be K_0 and assume a depreciation of 10%. To generate price data in this economy, we use the valuation equation implied by the neoclassical model, that is:

$$VR_{it} = (1 + (1-\tau_t)\theta^2 \frac{I_t}{K_{t-1}})K_{t-1} \tag{B.2}$$

where $VR_{it} \equiv \frac{P_{it}}{K_{it}}$ in which P_{it} is the market value of equity. Thus, by construction, the observed and the model-implied valuation ratio are equal.

The econometric exercise of interest here is to investigate the extent to which the different estimation approaces allow us to recover the structural parameters, which in our case is the parameter θ (we ignore the estimation of the curvature parameter here for simplicity). To make the results more general, we consider three set the slope adjustment cost parameters $\theta = 10$, 20, or

40. The curvature is fixed at 2 (quadratic). Given these parameters, we can generate a time series of valuation ratios in the model using equations (B.2).

To examine the role of the impact of portfolio-level aggregation of the characteristics using the LWZ procedure, we first create 10 and 50 portfolios sorted on the firm-level lagged valuation ratio (VR) and investment-rate (IK). As in LWZ, we construct the portfolio-level counterpart of the valuation ratio as follows. For each portfolio $j = 1, ..., 10$, or 50 , and in each period, we have:

$$VR_{jt} = \frac{\sum_i^N P_{it}}{\sum_i^N K_{it}} \text{ , i} \in \text{Portfolio j} \tag{B.3}$$

$$I_{jt}/K_{jt-1} = \frac{\sum_i^N I_{it}}{\sum_i^N K_{it-1}} \tag{B.4}$$

To estimate the model parameters we construct the model-implied predicted valuation ratio $\widehat{VR}_{jt}$ as:

$$\widehat{VR}_{jt} \equiv (1 + (1 - \tau_t)\hat{\theta}^2 \frac{I_t}{K_{t-1}})K_{t-1}$$

which uses the portfolio-level investment rate computed as in equation (B.4). Following LWZ, we estimate the model parameters (θ) by the Generalized Method of Moments (GMM) using the moment condition:

$$E\left[VR_{jt} - \widehat{VR}_{jt}\right] = 0,\, j = 1, .., 10 \textit{ or } 50 \tag{B.5}$$

We use the identity matrix as the weighting matrix. We label this method as GMM-XS. For comparison with the estimation approach used here that matches the time series data (and to establish that the conclusions here do not depend on the estimation approach used), we also estimate the parameters by minimizing the sum of squared residuals. That is, define:

$$\varepsilon_{jt} = VR_{jt} - \widehat{VR}_{jt},$$

and then estimate the model parameters using the first order conditions from the minimization of $\sum_{t=1}^T \sum_{j=1}^N \varepsilon_{jt}^2$. We denote this method as NLLS -TS. For each estimation method, we report the parameter estimate of the slope coefficient θ (reported as $\hat{\theta}$) for the three cases θ = 10, 20, or 40, together with the estimation bias, computed as the percentage deviation of the estimated parameter value relative to the true parameter value (bias= $\frac{\hat{\theta}-\theta}{\theta}$).

[Insert Table 8 here]

Table 8, rows LWZ, report the estimation results using the LWZ aggregation method, XSMED, reports results with our aggregation using the median and XSEW uses the equality weighted mean. Panel A reports the results using the 10 valuation ratio (VR) portfolios, and Panel B reports the results using 50. Panel C reports the results using the 10 investment rate (IK) portfolios, and Panel D reports the results using 50. The columns in the right report the results using the GMM-XS estimation approach (that is, matching the cross section- average- of each series) , while the columns at the left report the results using NLLS-TS estimation approach (that is, matching the time series of the series).

Table 8 reveals that across all cases, the parameter estimates using the LWZ aggregation procedure differ from the true firm-level structural parameters, and hence do not have a structural interpretation. In all cases considered here, the bias in the estimation range from -73.84% to 8.52%, and its never zero. Also, the parameter estimates vary significantly across the set of test assets used for the estimation (IK or VR portfolios), across the number of portfolios (10 vs 50) and across estimation procedure (GMM-XS vs NLLS-TS), which should not occur in large samples if the estimation procedure is consistent, in which case the procedure should recover the true underlying parameter value. Indeed, the variation of the parameter estimates across test assets helps us understand why the parameter estimates in LWZ vary significantly across different test assets used in the estimation. The bias occurs here because of aggregation issues in the procedure. The nonlinearities in the valuation ratio mean that the true portfolio-level valuation ratio is different from the portfolio-level valuation ratio obtained by first aggregating each portfolio-level characteristics (investment rate, etc) separately, to construct the portfolio-level valuation ratio counterparts. A larger number of portfolios and a estimation procedure that takes the time-series into account minimizes the bias because it decreases aggregation.

Turning to the analysis of the alternative estimation procedures discussed in the main text, namely cross sectional mean aggregation (XSMED) and equal-weighted aggregation (XSEW), Table 8 shows that these methods avoid the aggregation issues in LWZ. In particular, the results in Table 8 show that the three alternative aggregation procedures are unbiased, thus allowing us to recover the true underlying firm-level structural parameters.

Naturally, with measurement error, the analysis becomes significantly more complicated. Since measurement error in firm-level data is not directly observed, different assumptions about the nature of the error may lead to different results. This does not invalidate the previous analysis. The analysis here shows that even without measurement error, the aggregation procedure in LWZ contaminates the parameter estimates, which in turn invalidates the interpretation of the parameter estimates as firm-level structural parameters. While its theoretically possible that measurement error in the data might lead any inconsistent estimation method to recover the true parameter value in the data, this is unlikely to be case here, especially when a large set of moments and a large set of test assets is used in the estimation.

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Figures and Tables

Table 1 : Summary Statistics

Panel A reports the time-series average of the cross-sectional median, and the standard-deviation of selected characteristics of the firm level data.VR$_{it}$ is the firm's valuation ratio. I_{it}^P/K_{it}^P is investment rate in physical capital, H_{it}/L_{it} is investment rate in labor stock (hiring rate), I_{it}^K/U_t^K is investment rate in knowledge capital and I_{it}^B/U_t^B is investment rate in brand capital. We also present the stock of each input (physical capital, labor, knowledge capital and brand capital) relative to the sum of the three capital inputs. Panel B, shows cross-correlations of the investment/hiring rates.

Panel A: Descriptive Statistics

	Median			S.D.		
	All	Low S.	High S.	All	Low S.	High S.
	Valuation ratios					
VR_{it}	1.97	1.31	2.10	3.17	2.06	3.29
	Investment/hiring rates					
I_{it}^P/K_{it}^P	0.17	0.13	0.18	0.25	0.17	0.26
H_{it}/L_{it}	0.16	0.15	0.17	0.25	0.22	0.26
I_{it}^K/K_{it}^K	0.27	0.20	0.29	0.21	0.13	0.21
I_{it}^B/K_{it}^B	0.26	0.25	0.26	0.20	0.15	0.21
	Scaled capital and labor ratios					
K_{it}^P/A_{it}	0.44	0.64	0.41	0.23	0.21	0.22
$(W_{it-1}L_{it})/A_{it}$	0.28	0.25	0.29	0.29	0.19	0.31
U_{it}^K/A_{it}	0.11	0.16	0.10	0.15	0.18	0.14
U_{it}^B/A_{it}	0.36	0.13	0.42	0.24	0.14	0.23

Panel B: Correlations - across all firms

	H_{it}/L_{it}	I_{it}^K/K_{it}^K	I_{it}^B/K_{it}^B
I_{it}^P/K_{it}^P	0.42	0.48	0.47
H_{it}/L_{it}	–	0.31	0.37
I_{it}^K/K_{it}^K	–	–	0.42

Table 2 : Firm Value Decomposition Based on Book Values

This table reports the fraction of firm value that is attributed to each input (μ) based on its book value. This decomposition is done by setting all the adjustment costs to zero and evaluating at the median value of the ratio of the capital inputs. The results are reported for all firms, low and high skill industries.

	All	Low S.	High S.
$\bar{\mu}^P$: Physical capital	60.20	78.10	56.02
$\bar{\mu}^L$: Labor	0.00	0.00	0.00
$\bar{\mu}^K$: Knowledge capital	9.32	12.09	8.46
$\bar{\mu}^B$: Brand capital	30.49	9.82	35.52

Table 3 : Baseline Estimation and Alternative Model Specifications

This table reports estimation results, measures of fit and the implied firm value decomposition. The columns show the values for different model specifications. The estimation uses 10 portfolios for each of the 4 sorting variables. The estimation is done using the cross-sectional median aggregation method and NLLS methodology. Panel A reports the estimation results.θ_P, θ_L, θ_K and θ_B are respectively, the physical capital, labor, knowledge capital and brand capital slope adjustment cost parameter. ν_P, ν_L, ν_K and ν_B are, respectively, the physical capital, labor, knowledge capital and brand capital curvature adjustment cost parameters. s.e. stands for bootstraped standard errors. Panel B reports measures of fit and value decomposition. m.a.e. is the mean absolute error of the valuation error, and m.a.e./$|VR|$ is the mean absolute valuation error scaled by the absolute value of the ratio. The table reports the median fraction of the value that is attributed to each input (μ) . CX/Y is the ratio (in percent) of the implied input adjustment costs-to-sales ratio.

Panel A: Parameter estimates

Column number:	(1)	(2)	(3)	(4)	(5)
Slope					
θ_P	1.94	3.24	2.45	3.09	3.62
s.e.	[0.51]	[0.45]	[0.29]	[0.34]	[0.44]
θ_L	3.45		4.23		
s.e.	[0.30]		[0.24]		
θ_K	1.90			2.98	
s.e.	[0.51]			[0.26]	
θ_B	3.27				4.38
s.e.	[0.33]				[0.18]
Curvature					
ν_P	2.50	3.89	3.33	3.77	2.67
s.e.	[0.73]	[0.37]	[0.50]	[0.49]	[0.65]
ν_L	1.64		1.50		
s.e.	[0.18]		[0.18]		
ν_K	1.99			2.81	
s.e.	[0.67]			[0.52]	
ν_B	2.09				1.37
s.e.	[0.33]				[0.21]
Intercept	−0.34	1.37	0.49	1.08	−0.21
s.e.	[0.26]	[0.07]	[0.07]	[0.06]	[0.11]

Table 3 : Baseline Estimation and Alternative Model Specifications (cont.)

Panel B: Measures of fit, implied firm value decomposition, and adjustment costs

Column number:	(1)	(2)	(3)	(4)	(5)
	Measures of fit				
XS- R^2	0.94	0.64	0.76	0.72	0.64
TS- R^2	0.71	0.09	0.53	0.14	0.46
m.a.e.	0.41	0.78	0.53	0.76	0.53
m.a.e./$\mid VR \mid$	0.19	0.36	0.24	0.35	0.25
	Firm value decomposition (in %)				
$\bar{\mu}^P$: Physical capital	29.66	100.00	43.40	77.50	46.90
$\bar{\mu}^L$: Labor	27.21	–	56.60	–	–
$\bar{\mu}^K$: Knowledge capital	6.06	–	–	22.50	–
$\bar{\mu}^B$: Brand capital	37.07	–	–	–	53.10
	Adjustment costs (in %)				
CP/Y : Physical capital	0.61	0.72	0.44	0.65	2.39
CL/Y : Labor	8.93	–	13.93	–	–
CK/Y : Knowledge capital	3.14	–	–	4.70	–
CB/Y : Brand capital	1.77	–	–	–	4.33

Table 4 : Estimation Across Labor Skill Industries

This table reports estimation results, measures of fit and the implied firm value decomposition. The columns show the values for different model specifications for low, median and high labor skill. The estimation uses 10 portfolios for each of the 4 sorting variables. The estimation is done using the cross-sectional median aggregation method and NLLS methodology. Panel A reports the estimation results.θ_P, θ_L, θ_K and θ_B are respectively, the physical capital, labor, knowledge capital and brand capital slope adjustment cost parameter. ν_P, ν_L, ν_K and ν_B are, respectively, the physical capital, labor, knowledge capital and brand capital curvature adjustment cost parameters. s.e. stands for bootstraped standard errors. Panel B reports measures of fit and value decomposition. m.a.e. is the mean absolute error of the valuation error, and m.a.e./$|VR|$ is the mean absolute valuation error scaled by the absolute value of the ratio. The table reports the median fraction of the value that is attributed to each input (μ) . CX/Y is the ratio (in percent) of the implied input adjustment costs-to-sales ratio.

Panel A: Parameter estimates

Column number:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Skill:	L	M	H	L	H	L	H	L	H	L	H
Slope											
θ_P	3.42	2.37	1.77	2.92	3.00	2.3	1.88	3.16	2.54	2.97	3.38
s.e.	[1.07]	[0.86]	[0.56]	[0.97]	[1.07]	[1.96]	[1.22]	[0.93]	[0.67]	[0.60]	[0.55]
θ_L	1.70	2.77	3.38			1.91	4.2				
s.e.	[0.55]	[0.42]	[0.31]			[1.92]	[0.78]				
θ_K	4.12	2.71	1.86					3.50	2.81		
s.e.	[0.78]	[0.79]	[0.64]					[0.58]	[0.35]		
θ_B	4.83	4.45	3.05							2.46	4.63
s.e.	[2.84]	[1.33]	[0.45]							[1.43]	[0.41]
Curvature											
ν_P	1.52	2.53	2.67	2.49	3.56	2.68	3.38	2.14	4.15	2.74	2.53
s.e.	[1.76]	[1.21]	[1.73]	[2.46]	[0.83]	[2.41]	[2.78]	[2.54]	[0.87]	[0.81]	[0.63]
ν_L	1.70	1.27	1.52			1.87	1.85				
s.e.	[1.72]	[0.32]	[0.17]			[2.20]	[2.04]				
ν_K	2.23	1.98	2.31					2.61	3.41		
s.e.	[0.91]	[0.53]	[1.93]					[0.98]	[1.01]		
ν_B	1.10	1.27	2.52							1.10	1.47
s.e.	[2.82]	[0.16]	[0.37]							[1.28]	[0.16]
Intercept	− 1.14	− 0.65	− 0.19	0.81	1.64	0.72	0.87	0.11	1.46	0.58	− 0.35
s.e.	[0.60]	[0.55]	[0.30]	[0.23]	[0.14]	[0.67]	[0.44]	[0.35]	[0.11]	[0.20]	[0.34]

Table 4 : Estimation Across Labor Skill Industries (cont.)

Panel B: Measures of fit, implied firm value decomposition, and adjustment costs

Column number:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Skill:	L	M	H	L	H	L	H	L	H	L	H
	Measures of fit										
XS-R^2	0.74	0.89	0.94	0.30	0.58	0.26	0.78	0.47	0.70	0.23	0.63
TS-R^2	0.38	0.56	0.67	0.02	0.07	0.10	0.52	0.26	0.13	0.03	0.41
m.a.e.	0.67	0.47	0.47	0.87	0.85	0.83	0.57	0.76	0.82	0.73	0.61
m.a.e./$\mid VR \mid$	0.37	0.25	0.2	0.48	0.36	0.45	0.24	0.42	0.34	0.41	0.26
	Firm value decompostion (in %)										
$\bar{\mu}^P$: Physical capital	60.48	38.62	24.92	100.00	100.00	82.63	39.98	76.47	74.76	85.53	35.73
$\bar{\mu}^L$: Labor	6.61	22.75	29.91	–	–	17.37	60.02	–		–	–
$\bar{\mu}^K$: Knowledge capital	18.83	9.57	5.28	–	–	–		23.53	25.24	–	–
$\bar{\mu}^B$: Brand capital	14.08	29.06	39.89	–	–	–	–	–	–	14.47	64.27
	Adjustment costs (in %)										
CP/Y : Physical capital	5.30	0.67	0.50	1.06	1.02	0.44	0.24	1.99	0.34	0.74	3.01
CL/Y : Labor	1.56	7.48	11.47			1.44	12.46	–	–	–	–
CK/Y : Knowledge capital	1.04	1.46	3.47	–	–	–	–	0.55	5.36	–	–
CB/Y : Brand capital	7.62	4.95	1.21	–	–	–	–	–	–	3.58	4.36

Table 5 : Decomposing Firm Value Across Decades

The table reports the average value of the median fraction of the value that is attributed to each input over different decades. The calculations are done using the estimates of Table 3 column (1) for all firms and 4 column (1)-(3) for each labor skill.

	$70s$	$80s$	$90s$	$00s$	$10s$
			All Firms		
$\bar{\mu}^P$: Physical capital	48.64	40.23	24.53	19.31	18.24
$\bar{\mu}^L$: Labor	20.43	20.17	28.24	32.90	36.44
$\bar{\mu}^K$: Knowledge capital	7.63	7.14	6.42	4.67	3.95
$\bar{\mu}^B$: Brand capital	23.29	32.46	40.80	43.12	41.37
			Low Skill		
$\bar{\mu}^P$: Physical capital	70.29	65.55	59.24	52.47	58.64
$\bar{\mu}^L$: Labor	3.90	5.34	6.58	8.23	9.18
$\bar{\mu}^K$: Knowledge capital	16.11	14.82	17.47	23.55	23.85
$\bar{\mu}^B$: Brand capital	9.69	14.28	16.71	15.75	8.33
			Mid Skill		
$\bar{\mu}^P$: Physical capital	51.59	47.09	35.80	30.51	28.55
$\bar{\mu}^L$: Labor	16.57	16.43	25.82	26.30	29.69
$\bar{\mu}^K$: Knowledge capital	9.32	9.95	10.14	9.61	7.46
$\bar{\mu}^B$: Brand capital	22.52	26.54	28.24	33.59	34.30
			High Skill		
$\bar{\mu}^P$: Physical capital	40.87	34.54	21.02	15.83	13.41
$\bar{\mu}^L$: Labor	23.33	22.76	29.89	35.97	40.90
$\bar{\mu}^K$: Knowledge capital	6.49	6.40	6.00	3.83	2.78
$\bar{\mu}^B$: Brand capital	29.31	36.30	43.10	44.37	42.90

Table 6 : Business Cycle Properties of the Value and Shares of the Capital and Labor Inputs

This table reports the cyclicality of the value ratio and the cyclicality and correlation of each component of the decomposition. The cyclical component is calculated using HP-filter with smoothing factor of 100 on the log each portfolio time series and on the HP-filtered log of the aggregate sales time series. The correlations are also calculated using the hp filtered series. Panel A displays the cyclicality of the value ratio (VR) , the shares (μ) and value components (V) and the standard deviation of each series for all firms and high and low skill. Panel B displays the cross input correlations of the shares and value components.

	Cyclicality Correl. with Y^{agg}			S.D.		
Industry:	All	Low S.	High S.	All	Low S.	High S.
VR : Valuation ratio	0.41	0.15	0.37	0.10	0.06	0.11
	Capital/labor shares (HP cycle)					
$\bar{\mu}^P$: Physical capital	−0.45	0.00	−0.51	0.09	0.04	0.09
$\bar{\mu}^L$: Labor	0.25	0.04	0.31	0.25	0.20	0.15
$\bar{\mu}^K$: Knowledge capital	−0.28	0.23	−0.17	0.09	0.12	0.08
$\bar{\mu}^B$: Brand capital	−0.46	−0.39	−0.29	0.06	0.15	0.07
	Capital/labor values (HP cycle)					
V^P : Physical capital	0.09	0.13	−0.15	0.04	0.07	0.05
V^L : Labor	0.33	0.07	0.36	0.31	0.20	0.21
V^K : Knowledge capital	0.35	0.25	0.40	0.05	0.15	0.06
V^B : Brand capital	0.19	−0.37	0.22	0.07	0.14	0.09

Table 7 : Robustness Checks

This table reports the estimation results and measures of fit and value decomposition. Panel A reports the estimation results.θ_P, θ_L, θ_K and θ_B are respectively, the physical capital, labor, knowledge capital and brand capital slope adjustment cost parameter. ν_P, ν_L, ν_K and ν_B are, respectively, the physical capital, labor, knowledge capital and brand capital curvature adjustment cost parameters. s.e. stands for bootstraped standard errors.Panel B reports measures of fit and value decomposition. m.a.e. is the mean absolute error of the valuation error, and m.a.e./$|VR|$ is the mean absolute valuation error scaled by the absolute value of the ratio. The columns show values for different aggregation methods and for different number of portfolios. XSMED is the cross-sectional median aggregation method and XSEW is the cross-sectional equal-weighted aggregation method. VR_{75-25} minimizes the difference between the estimated error of quantile 75 and the quantile 25 while $VR_{25,50,75}$ minimizes the estimation error of quantiles 25, 50 and 75. Finally, FL performs the estimation using firm level data. Panel B reports measures of fit and value decomposition. m.a.e. is the mean absolute error of the valuation error, and m.a.e./$|VR|$ is the mean absolute valuation error scaled by the absolute value of the ratio. The table reports the median fraction of the value that is attributed to each input (μ) . CX/Y is the ratio (in percent) of the implied input adjustment costs-to-sales ratio.

Panel A: Parameter estimates

	XSMED				XSEW	VR_{75-25}	$VR_{25,50,75}$	FL
Column number:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Number of portfolios per sort:	10	2	5	20	10	10	10	–
Total number of portfolios:	40	8	20	80	40	40	40	–
Slope								
θ_P	1.94	1.46	1.91	1.97	1.83	2.32	1.63	2.00
s.e.	[0.51]	[0.34]	[0.42]	[0.54]	[0.56]	[1.90]	[0.50]	[0.56]
θ_L	3.45	4.07	3.71	3.20	3.98	3.99	3.80	2.19
s.e.	[0.30]	[0.20]	[0.23]	[0.33]	[0.18]	[0.74]	[0.38]	[0.15]
θ_K	1.90	1.61	1.98	2.00	2.66	2.44	1.70	2.92
s.e.	[0.51]	[0.42]	[0.34]	[0.50]	[0.43]	[1.72]	[0.57]	[0.47]
θ_B	3.27	3.01	3.12	3.31	3.76	3.18	3.16	4.18
s.e.	[0.33]	[0.19]	[0.25]	[0.42]	[0.38]	[2.17]	[0.37]	[0.56]
Curvature								
ν_P	2.50	2.72	2.27	2.84	3.59	2.54	2.70	2.66
s.e.	[0.73]	[0.45]	[0.88]	[0.45]	[1.31]	[1.20]	[0.35]	[0.71]
ν_L	1.64	1.64	1.49	1.35	1.28	1.60	1.60	1.42
s.e.	[0.18]	[0.18]	[0.17]	[0.17]	[0.08]	[0.29]	[0.19]	[0.10]
ν_K	1.99	2.10	2.04	2.26	1.99	2.15	2.12	2.43
s.e.	[0.67]	[0.49]	[0.68]	[0.69]	[0.63]	[0.78]	[0.56]	[0.97]
ν_B	2.09	2.24	2.15	2.11	1.95	2.55	2.69	1.55
s.e.	[0.33]	[0.33]	[0.31]	[0.24]	[0.14]	[1.56]	[0.25]	[0.22]
Intercept	−0.34	−0.33	−0.44	−0.35	−0.49	0.50	−0.28	0.15
s.e.	[0.26]	[0.12]	[0.15]	[0.34]	[0.27]	[0.11]	[0.26]	[0.32]

Table 7 : Robustness Checks (cont.)

Panel B: Measures of fit, implied firm value decomposition, and adjustment costs

	XSMED				XSEW	VR_{75-25}	$VR_{25,50,75}$	FL
Column number:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Number of portfolios per sort:	10	2	5	20	10	10	10	–
Total number of portfolios:	40	8	20	80	40	40	40	–
	Measures of fit							
XS- R^2	0.94	0.95	0.95	0.93	0.96	–	0.92	–
TS- R^2	0.71	0.78	0.76	0.66	0.76	–	0.69	0.26
m.a.e.	0.41	0.31	0.37	0.46	0.50	3.43	0.42	2.00
m.a.e./$\mid VR \mid$	0.19	0.15	0.17	0.21	0.17	1.10	0.19	0.60
	Firm value decomposition (in %)							
$\bar{\mu}^P$: Physical capital	29.66	26.59	29.29	28.32	21.99	30.06	27.98	26.23
$\bar{\mu}^L$: Labor	27.21	35.90	31.71	28.58	32.66	32.38	33.61	14.57
$\bar{\mu}^K$: Knowledge capital	6.06	5.16	5.98	5.93	7.30	7.15	5.48	8.71
$\bar{\mu}^B$: Brand capital	37.07	32.35	33.02	37.17	38.04	30.41	32.92	50.48
	Adjustment costs (in %)							
CP/Y : Physical capital	0.61	0.21	0.83	0.40	0.11	0.00	0.00	0.59
CL/Y : Labor	8.93	11.77	11.67	11.03	15.85	0.92	0.29	6.28
CK/Y : Knowledge capital	3.14	1.96	3.21	2.63	6.08	11.72	10.91	6.14
CB/Y : Brand capital	1.77	1.35	1.55	1.79	2.49	4.51	2.15	3.81

Table 8 : Comparison of Estimation Methods: the Impact of Portfolio-Level Aggregation

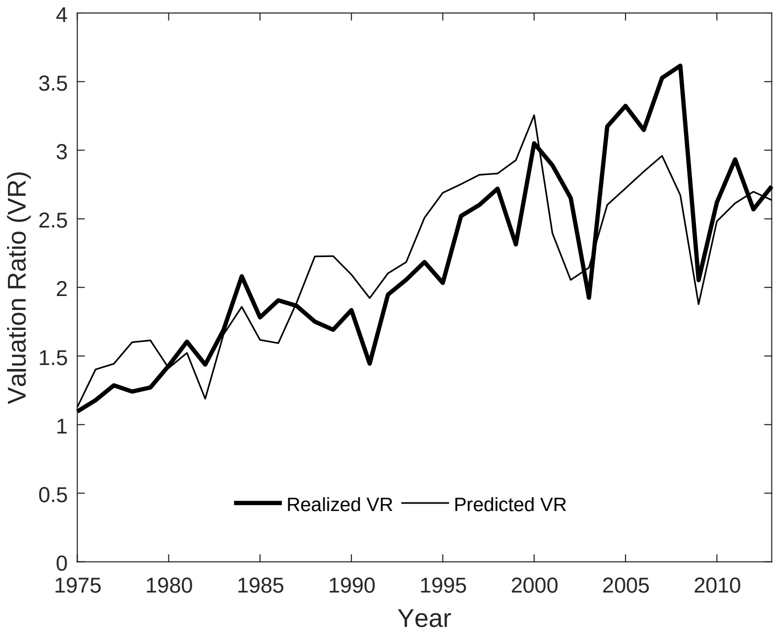
This table reports the estimates of the model parameters across different portfolio-level aggregation methods for the physical capital only model with curvature equal to 2, the slope is represented by beta as we describe in Section B. We consider three set of true model parameters at the firm level: $\theta = 10$, $\theta = 20$, or $\theta = 40$. For each method, $\hat{\theta}$ is the estimated parameter, and bias is the percentage deviation of the estimated parameter value relative to the true parameter value (bias= $\frac{\hat{\theta}-\theta}{\theta}$)). In LWZ the data is aggregated by first aggregating the characteristics to obtain the portfolio-level predicted valuation ratio as described in Section B. XSMED is the cross-sectional median aggregation method in which we compute the portfolio-level observed and predicted cross sectional median of the valuation ratio across all the firms in the portfolios in each year; XSEW is the equal-weighted cross sectional mean aggregation method in which we compute the portfolio-level observed and predicted cross sectional valuation ratio across all the firms in the portfolios in each year. In Panel A(B), the test assets are 10(50) value ratio portfolios, and in Panel C(D) the test assets are 10(50) investment rate portfolios. Two estimation methods are used. In NLLS-TS the parameters are obtained by minimizing the sum of squared portfolio-level residual (the difference between observed and model-implied valuation ratio) at the portfolio-level. In GMM-XS the parameters are obtained by matching the average observed and predicted valuation ratio of each portfolio (as in LWZ).

	NLLS-TS						GMM-XS					
True Value:	$\theta = 10$		$\theta = 20$		$\theta = 40$		$\theta = 10$		$\theta = 20$		$\theta = 40$	
Estimate:	$\hat{\theta}$	Bias (%)	$\hat{\theta}$	Bias (%)	$\hat{\theta}$	Bias (%)	$\hat{\theta}$	Bias (%)	$\hat{\theta}$	Bias (%)	$\hat{\theta}$	Bias (%)
Panel A: Estimation across 10 VR Portfolios												
LWZ/BXZ	6.86	− 31.36	13.89	−30.54	27.80	−30.51	2.62	−73.84	6.90	−65.52	14.47	−63.82
XSMED	10.00	0.00	20.00	0.00	40.00	0.00	10.00	0.00	20.00	0.00	40.00	0.00
XSEW	10.00	0.00	20.00	0.00	40.00	0.00	10.00	0.00	20.00	0.00	40.00	0.00
Panel B: Estimation across 50 VR Portfolios												
LWZ/BXZ	8.90	−10.97	18.30	−8.52	35.98	−10.05	5.28	−47.20	11.53	−42.36	23.02	−42.45
XSMED	10.00	0.00	20.00	0.00	40.00	0.00	10.00	0.00	20.00	0.00	40.00	0.00
XSEW	10.00	0.00	20.00	0.00	40.00	0.00	10.00	0.00	20.00	0.00	40.00	0.00
Panel C: Estimation across 10 IK Portfolios												
LWZ/BXZ	3.35	−66.48	8.01	−59.97	16.60	−58.50	2.48	−75.21	6.80	−65.98	14.38	−64.04
XSMED	10.00	0.00	20.00	0.00	40.00	0.00	10.00	0.00	20.00	0.00	40.00	0.00
XSEW	10.00	0.00	20.00	0.00	40.00	0.00	10.00	0.00	20.00	0.00	40.00	0.00
Panel D: Estimation across 50 IK Portfolios												
LWZ/BXZ	6.88	−31.21	14.15	−29.24	28.50	−28.76	5.88	−41.22	12.38	−38.11	25.06	−37.36
XSMED	10.00	0.00	20.00	0.00	40.00	0.00	10.00	0.00	20.00	0.00	40.00	0.00
XSEW	10.00	0.00	20.00	0.00	40.00	0.00	10.00	0.00	20.00	0.00	40.00	0.00

Figure 1 : Time-Series and Cross-Sectional Fit of the Baseline Model

Panel A plots the predicted versus realized time series average of the valuation ratio from the estimation of the investment-based model using the cross sectional median (XSMED) estimation method and 40 portfolios as test assets. Panel B, plots the average of the across time for each portfolio for the predicted and realized valuation ratio.

Panel A: Time-series fit



Panel B: Cross-sectional fit

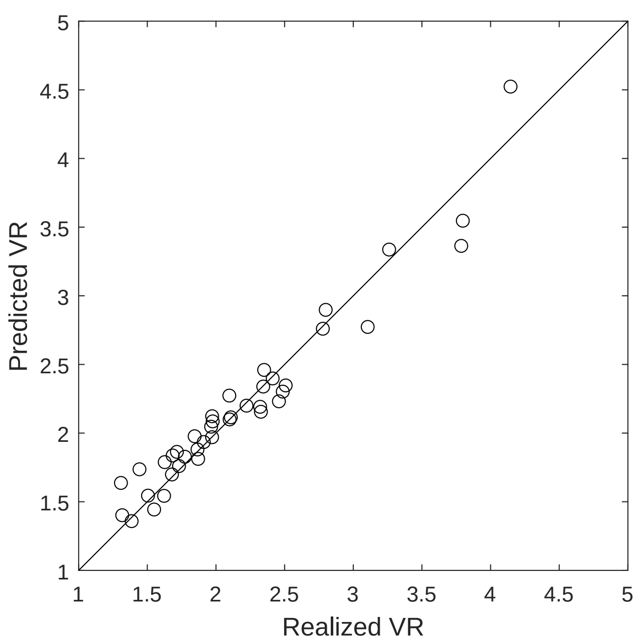


Figure 2 : Time-Series of the Baseline Model Across Portfolio Sorts

This figure plots the the predicted versus realized time series average of the median portfolio for each of the portfolio sorts.

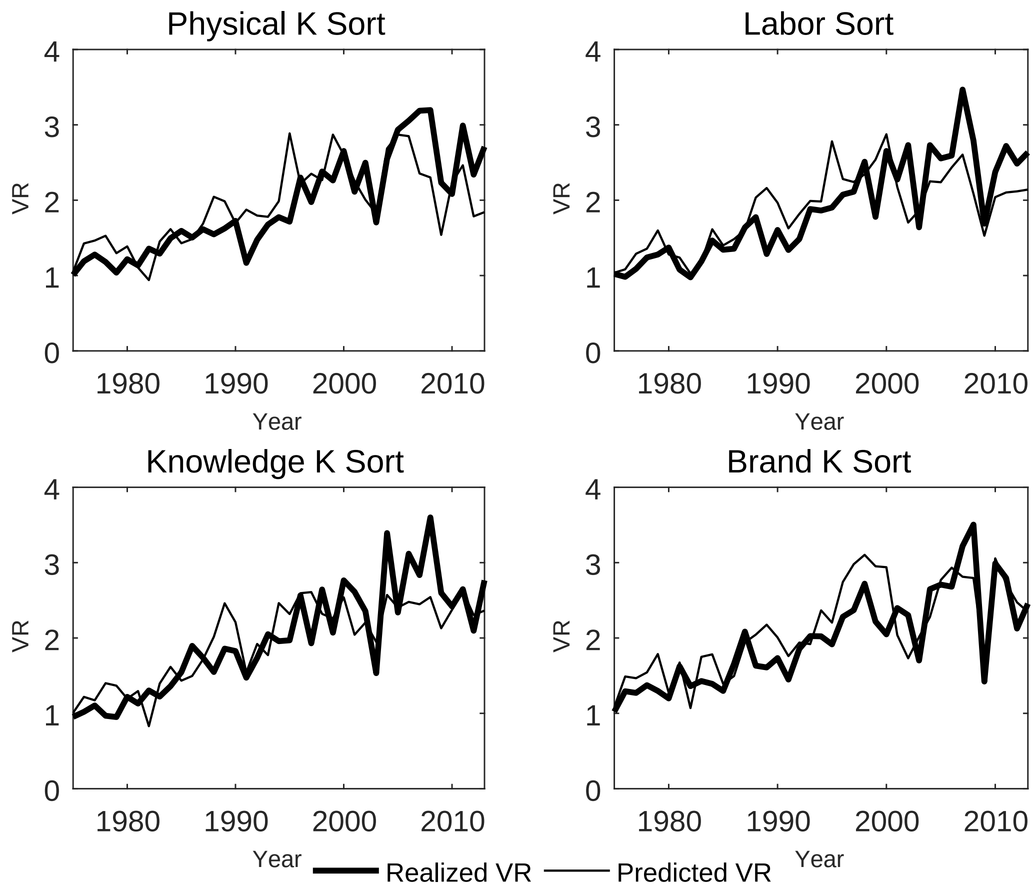
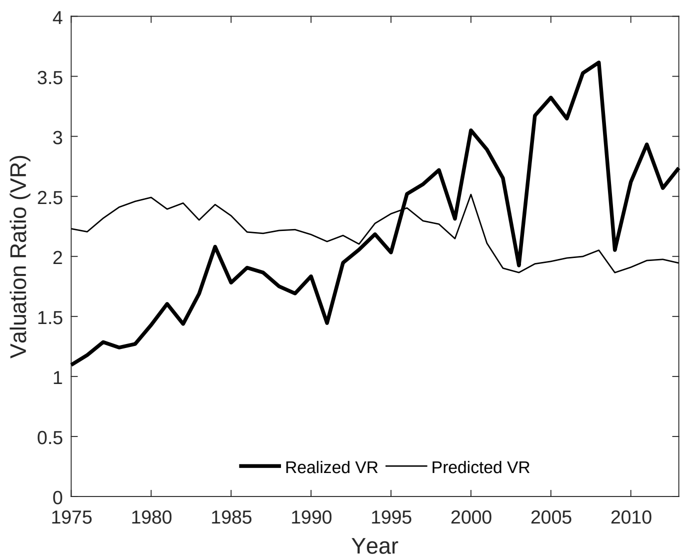


Figure 3 : Time-Series and Cross-Sectional Fit of the One-Capital Input Model

Panel A plots the predicted versus realized time series average of the valuation ratio from the estimation of the investment-based model using the cross sectional median (XSMED) estimation method and 40 portfolios as test assets. Panel B, plots the average of the across time for each portfolio for the predicted and realized valuation ratio.

Panel A: Time-series fit



Panel B: Cross-sectional fit

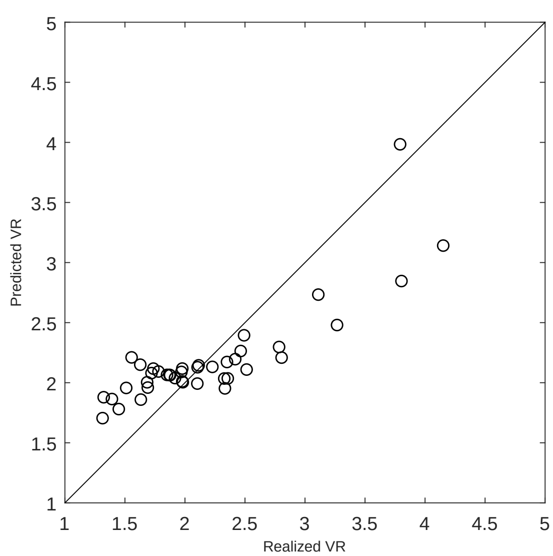
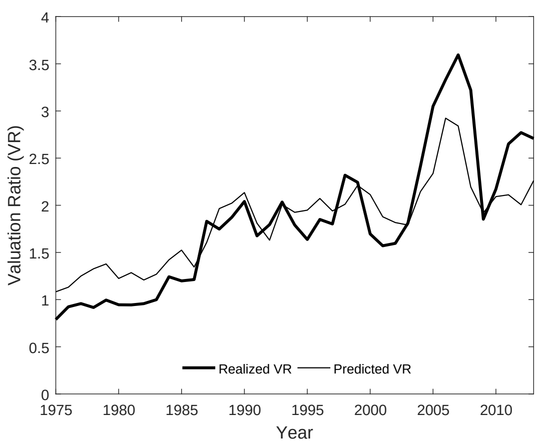


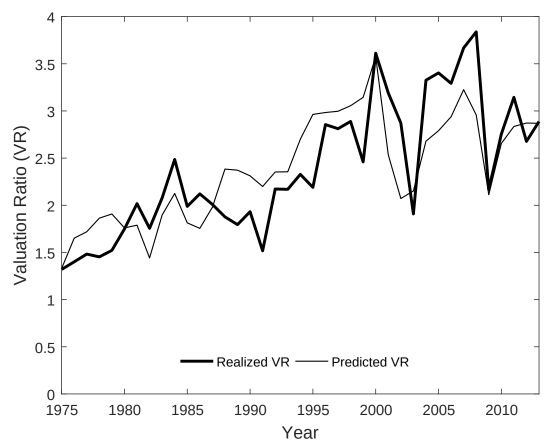
Figure 4 : Time-Series and Cross-Sectional Fit Across Industries

Panel A (C) plots the predicted versus realized time series average of the valuation ratio for low(high) skill from the estimation of the investment-based model using the cross sectional median (XSMED) estimation method and 40 portfolios as test assets. Panel B (D), plots the average of the across time for low(high) skil each portfolio for the predicted and realized valuation ratio.

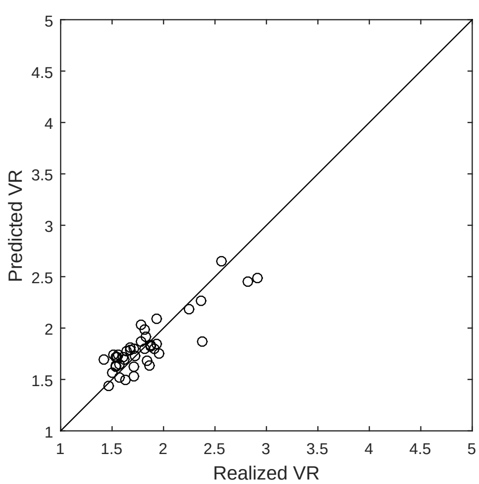
Panel A: Time-Series Fit, Low Skill



Panel C: Time-Series Fit, High Skill



Panel B: Cross-Sectional Fit, Low Skill



Panel D: Cross-Sectional Fit, High Skill

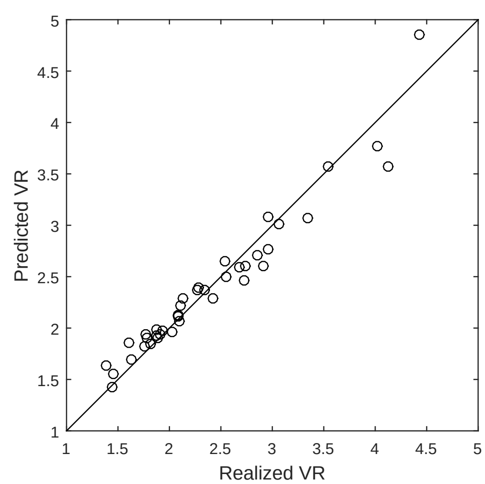
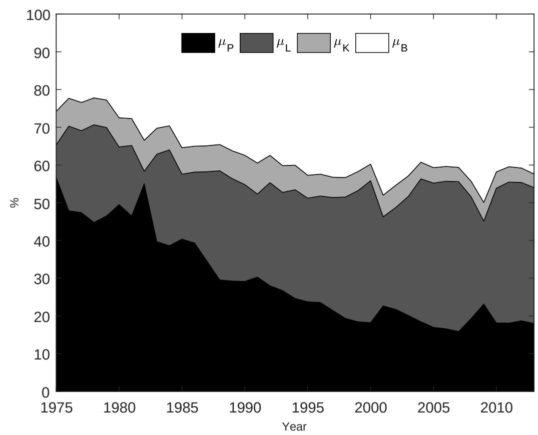


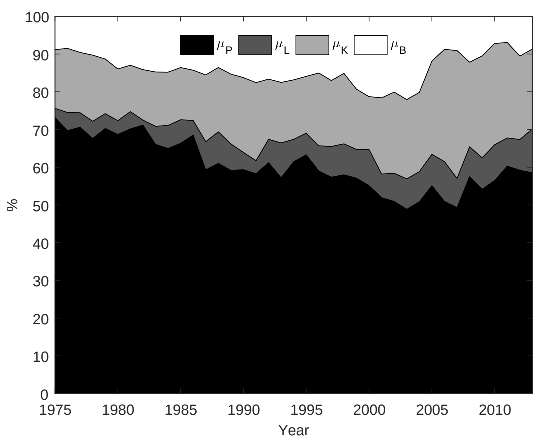
Figure 5 : Contribution of Each Input to Firm's Market Value Over Time

This figure plots the time series of the median contribution of each input for firms' market value (shares) implied by the estimation of the neoclassical investment model using the portfolio-level cross sectional median (XSMED) estimation method, and 40 portfolios as test assets. μ_P is the share of physical capital, μ_L is the share of labor, μ_K is the share of knowledge capital and μ_B is the share of brand capital. Panel A shows the results across all firms, Panel B shows the results across low labor-skill industries, and Panel C shows the results across high labor-skill industries.

Panel A: Shares all firms



Panel B: Shares low skill



Panel C: Shares high skill