Controlling inflation with timid monetary-fiscal regime changes*

Guido Ascari[a] — University of Oxford
Anna Florio[b] — Politecnico di Milano
Alessandro Gobbi[c] — Università di Pavia

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Abstract

Can monetary policy control inflation when both monetary and fiscal policies change over time? When monetary policy is active, a long-run fiscal principle entails flexibility in fiscal policy that preserves determinacy even when deviating from passive fiscal, substantially for brief periods or timidly for prolonged periods. To guarantee a unique equilibrium, monetary and fiscal policies must coordinate not only within but also across regimes, and not simply on being active or passive, but also on their extent. The amplitude of deviations from the active monetary/passive fiscal benchmark determines whether a regime is Ricardian: timid deviations do not imply wealth effects.

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[a]Corresponding author: Department of Economics, University of Oxford, Manor Road, Oxford OX1 3UQ, United Kingdom. E-mail address: guido.ascari@economics.ox.ac.uk

[b]Department of Management, Economics and Industrial Engineering, Politecnico di Milano, via Lambruschini 4/B, 20156 Milan, Italy. E-mail address: anna.florio@polimi.it

[c]Department of Economics and Finance, Università degli Studi di Pavia, via San Felice 5, 27100 Pavia, Italy. E-mail address: alessandro.gobbi@unicatt.it

1 Introduction

Under conventional views of price level determination, two conditions should be satisfied for monetary policy to be able to control inflation: the rational expectations equilibrium should be unique and Ricardian. The former is a desirable feature of monetary policy implementation because the presence of multiple stable equilibria would expose inflation (and output) to endogenous fluctuations; the latter ensures the absence of wealth effects from public debt dynamics that would foster spending and affect inflation. Both conditions are satisfied in the New Keynesian framework in which an active monetary and passive fiscal regime is assumed to operate.[1] In such a regime, monetary policy controls inflation while fiscal policy controls debt dynamics by adjusting to satisfy the government intertemporal budget constraint.

In addition to the standard active monetary/passive fiscal (AM/PF) regime, a passive monetary and active fiscal (PM/AF) regime also yields a unique solution. In this case, fiscal policy determines inflation because the price level must adjust to keep the real value of debt consistent with the government budget constraint. Hence, the so-called fiscal theory of the price level (FTPL) holds, where the absence of Ricardian equivalence produces wealth effects that, in turn, affect inflation.

Inflation dynamics are therefore a joint monetary-fiscal phenomenon. Moreover, expectations about future policies are crucial. The influential contribution by Davig and Leeper (2007) shows that a passive monetary policy—indeterminate in a static context—could return determinacy if monetary policy is expected to be sufficiently aggressive in the future. The authors conclude that this long-run Taylor principle dramatically expands the determinacy region relative to the constant-parameter setup. Consistent with much of the literature, Davig and Leeper (2007) place fiscal policy in the background by assuming a passive fiscal behaviour. However, as implied by the FTPL, agents' perceptions of whether government debt will bring about a higher tax burden in the future contribute to determine the inflation outcome. The ability of monetary policy to control inflation today depends on private sector beliefs about whether and how fiscal stress will be resolved in the future. More generally, the expectation of future regimes should be taken into account since *"interest rate policy, tax policy and expenditure policy, both now and as they are expected to evolve in the future, jointly determine the price level"* (Sims, 2016).

We address the long-standing question regarding the ability of monetary policy to control inflation,

[1] We apply the terminology in Leeper (1991). Active monetary (AM) policy arises when the response of the nominal interest rate to inflation is more than one-to-one. Otherwise we have passive monetary (PM) policy. Analogously, passive fiscal (PF) policy occurs when taxes respond sufficiently to debt to prevent its explosion; otherwise we have active fiscal (AF) policy.

in a setting where the monetary/fiscal mix can change over time.[2] The focus of our analysis will be primarily on the case in which one regime is AM/PF, which is the benchmark parameterisation in the New Keynesian literature.[3] The aim is to characterise the nature of all the possible different equilibria, then draw fresh policy implications and interpret the data through the lenses of our framework.

When agents' expectations incorporate the possibility of policy regime switches, the analysis of the uniqueness or multiplicity of rational expectations equilibria differs from the standard literature on determinacy/indeterminacy (e.g., Lubik and Schorfheide, 2004) in which the change in regime comes as a complete surprise and is perceived to last forever. It is possible that policy combinations that lead to an explosive or indeterminate equilibrium in a fixed-regime model do not do so in a Markov-switching model because agents anticipate the probability of reverting to a different policy mix in the future. We apply the perturbation method developed by Foerster et al. (2016, henceforth FRWZ) to a simple New Keynesian model with Markov-switching in monetary and fiscal policies. Despite the complexity of the solution algorithm under Markov-switching, we are able to provide some analytical insights into the nature of the solutions regarding both determinacy and implied inflation dynamics.

Our first result is to analytically define a *long-run fiscal principle*. Davig and Leeper's (2007) define a long-run Taylor principle to indicate the conditions that a switching monetary policy needs to satisfy to yield a unique rational expectations equilibrium in the Markov-switching framework, when assuming a passive fiscal policy. We symmetrically define a long-run fiscal principle to indicate the conditions that a switching fiscal policy needs to satisfy to yield a unique rational expectations equilibrium in the Markov-switching framework, when assuming an active monetary policy. As the long-run Taylor principle, the long-run fiscal principle entails some fiscal policy flexibility that could deviate from passive fiscal policy substantially for brief periods or timidly for prolonged periods. Our approach, therefore, extends Davig and Leeper's (2007) intuition to a model in which monetary and fiscal policy interact in determining inflation and economic dynamics. Using FRWZ's method, we were able to perform such an analysis in the presence of a state variable (i.e., government debt) and deliver some insightful analytical results.

Our second result is that, to yield determinacy, monetary and fiscal authorities should coordinate not only within regimes as suggested by Leeper (1991) but also *across regimes* by choosing the extent of activeness or passiveness. Our analysis leads to a natural generalisation of Leeper (1991) to a

[2] A growing body of literature uses models with recurring regime changes to estimate and study monetary and fiscal policy interactions. We mention several contributions without the aim of being exhaustive: Davig and Leeper (2006, 2007); Chung et al. (2007); Davig and Leeper (2008, 2011); Bianchi (2013); Bianchi and Melosi (2013); Foerster (2016); Bianchi and Ilut (2014); Leeper et al. (2015); Leeper and Leith (2016).

[3] The results, however, can easily be extended to any other regime combination.

Markov-switching context. Two cases are noteworthy. First, when one policy is substantially different across regimes, so should the other policy. A substantial deviation in both policies results in an overall switching policy mix. Second, when one policy is only timidly different across regimes, so should the other policy. A timid deviation into passive (or active) monetary (or fiscal) policy in one of the regimes returns a policy that is overall active (or passive) monetary (or fiscal) across the two regimes. In this case, we label the policy combination *overall active (or passive)*. In accordance with this new taxonomy, we show that monetary and fiscal policies need to be balanced across regimes to have a unique equilibrium: either an *overall switching policy mix*, where both policies are substantially switching, or an *overall AM/PF mix*, where an overall active monetary policy combination is coupled with an overall passive fiscal policy combination, and policies are allowed some flexibility up to timid deviations.

The third main result regards the nature of the solutions. Our taxonomy delivers a direct link between the concept of balanced policies and the presence of wealth effects or Ricardian dynamics. The literature usually refers to the AM/PF regime as Ricardian and to the PM/AF regime as non-Ricardian only when agents are assumed to be unaware of regime changes. In a model with recurrent regime changes, as Bianchi and Melosi (2013) note, the policy mix is insufficient to establish whether a regime is Ricardian. However, in our setup, the *overall AM/PF mix* allows a limited degree of flexibility for both monetary and fiscal policy and generates a unique Ricardian solution, with no wealth effects in either of the two regimes. Thus, our analysis establishes that, in a model in which agents are aware of recurrent regime changes, an *overall AM/PF mix* is definitively Ricardian. In our framework, the fact that agents assign a positive probability of moving towards a PM/AF regime in the future is not a sufficient condition to have wealth effects. Monetary and fiscal policy can timidly deviate from the standard new Keynesian AM/PF regime, and the central bank can still keep inflation under control, because there are no wealth effects arising from timid deviations from passive fiscal policy. In contrast, non-Ricardian dynamics prevail in an *overall switching mix*, that is, where the regime changes are sufficiently large.

Finally, our overall AM/PF regime is consistent with the case of a "timidity trap". Consider an unbacked fiscal expansion under a PM/AF regime, engineered to escape a liquidity trap. If the policy action is too timid, that is, if that PM/AF policy deviates only timidly from the previous AM/PF regime, it would not bring about the wealth effects needed to reflate the economy. To have the desired effects, there should be a clear departure from the previous regime, hence an overall switching mix. A BVAR on United States data for the recent ZLB period points towards this scenario: impulse response functions exhibits a deficit shock that is unable to spur inflation.

The paper is structured as follows. Section 2 introduces the model and methodology. Section 3 contains the main results and presents the long-run fiscal principle, our new taxonomy and the implications for policy coordination and for the dynamics of the model. Section 4 shows how our findings differ from those in earlier work on the same topic and how they can be applied to current economic issues such as ZLB policy. Section 5 concludes.

2 Model and methodology

2.1 The model

We consider a basic New Keynesian model with monetary and fiscal policy rules, as in Bhattarai et al. (2014). The model is well-known; thus, a more complete description is offered in the Appendix. In non-linear form, the equations of the model are the following:

$$1 = \beta E_t \left[\frac{Y_t - G}{Y_{t+1} - G} \frac{R_t}{\Pi_{t+1}} \right]; \tag{1}$$

$$\Omega_t \left(1 - \alpha \Pi_t^{\theta-1}\right)^{\frac{1}{1-\theta}} = \frac{(1-\alpha)^{\frac{1}{1-\theta}}}{1-\alpha} Y_t + \alpha\beta E_t \Pi_{t+1}^{\theta} \Omega_{t+1} \left(1 - \alpha \Pi_{t+1}^{\theta-1}\right)^{\frac{1}{1-\theta}}; \tag{2}$$

$$\Omega_t = \frac{Y_t}{Y_t - G} + \alpha\beta E_t \left[\Pi_{t+1}^{\theta-1} \Omega_{t+1} \right]; \tag{3}$$

$$\frac{b_t}{R_t} = \frac{b_{t-1}}{\Pi_t} + G - \tau_t; \tag{4}$$

$$\tau_t = \tau \left(\frac{b_{t-1}}{b} \right)^{\gamma_{\tau,t}} e^{u_{\tau,t}}; \tag{5}$$

$$R_t = R\left(\Pi_t\right)^{\phi_{\pi,t}} e^{u_{m,t}}. \tag{6}$$

Equation (1) is a standard Euler equation for consumption, where Y_t is output, R_t the nominal interest rate, Π_t the gross inflation rate and G government spending, which is assumed to be exogenous and constant. Equations (2) and (3) describe the evolution of inflation in the non-linear model. Ω_t is an auxiliary variable (equal to the present discounted value of expected future marginal revenues) that allows us to write the model recursively. Equation (4) is the government's flow budget constraint, where $b_t = B_t/P_t$ is real government debt. We follow Leeper (1991) in using lump-sum taxes (τ_t), which are set according to the fiscal rule (5): taxes react to the deviation of lagged real debt from its steady-state level (b) according to the parameter $\gamma_{\tau,t}$. Equation (6) describes monetary policy. It is a simple Taylor rule whereby the central bank reacts to current inflation according to the parameter

$_{;t}$. A variable without the time index (i.e., , b and R) indicates the value at the steady state. is the intertemporal discount factor; is the Dixit-Stiglitz elasticity of substitution between goods; and is the Calvo probability that a rm is unable to optimise its price.

The key parameters of our analysis are $_{;t}$ and $_{;t}$, which describe the time-varying stance of monetary and scal policy, respectively. We assume that these parameters follow an underlying two-state Markov process and are equal to ($_{;i}$, $_{;i}$) when the economy is in regime i, for $i = 1;2$. The transition probabilities of going from regime i to regime j are denoted by p_{ij} . Thus, p_{ii} is the probability of remaining in regime i, and $p_{ij} = 1 \quad p_{ii}$.

2.2 Solution method

As our model includes scal policy, we need to account for the dynamics of public debt, which is a state variable. We thus employ the perturbation method developed by FRWZ, the logic of which is analogous to an undetermined coe cient method applied to a Markov-switching context. It allows us to solve for all the minimal state variable (MSV) solutions of a Markov-switching model in the presence of predetermined variables.[4] Following FRWZ, our model can be written as follows:

$$E_t f\ (y_{t+1}; y_t; b_t; b_{t\ \ 1}; "_{t+1}; "_t; \ _{t+1}; \ _t) = 0; \tag{7}$$

where b_t is the only predetermined variable, while the remaining non-predetermined variables are stacked in vector $y_t^0 \quad [Y_t; \ _t; \ _t]$. The exogenous shocks appear in vector $"_t^0 \quad [u_{m;t}; u_{\ ;t}]$, and $\ _t^0 \quad [\ _{;t}; \ _{;t}]$ is the vector of Markov-switching parameters. The rst-order Taylor expansions of the recursive solutions are

$$b_t \quad b + h_i (b_{t\ \ 1} \quad b) + h_{i;"} "_t + h_{i;} \ \ ; \tag{8}$$

$$y_t \quad y + g_{i;b} (b_{t\ \ 1} \quad b) + g_{i;"} "_t + g_{i;} \ \ ; \tag{9}$$

for $i = 1;2$, where is the perturbation parameter. Note that the slope coe cients of the solutions are regime-dependent, while the steady state is not. The coe cients h_1 and h_2 govern the stability properties of the solution and are therefore the main focus in the analysis of determinacy. FRWZ show that h_i and $g_{i;b}$ can be jointly found after solving a system of quadratic equations. As this system

[4] Hence, by using this method, we consider only MSV solutions. While some other non-MSV solutions may still exist, the class of MSV solutions is usually that employed in the estimation of DSGE models. At the time of this writing, the analysis of rational expectation solutions in a Markov-switching context is a very active research area. In addition to FRWZ, see, among others, Farmer et al. (2009, 2011), Blake and Zampolli (2011), Cho (2016), Maih (2015), Barthelemy and Marx (2015).

cannot be solved using traditional approaches such as the generalised Schur decomposition, we follow FRWZ and adopt the Groebner basis algorithm to nd all existing MSV solutions.

Once all the solutions belonging to the MSV class of equilibria have been found, a stability criterion needs to be imposed to select the stable ones. We use the concept of mean square stability (MSS) proposed by Costa et al. (2005) and Farmer et al. (2009)[5]. The MSS condition constrains the values of the autoregressive roots in the state variable policy function in the two regimes. In the Appendix, we show that the necessary and su cient conditions for MSS are

$$(p_{11} + p_{22} \quad 1)\, h_1^2 h_2^2 < 1; \tag{10}$$

$$p_{11} h_1^2 \;\; 1 \quad h_2^2 \;\; + p_{22} h_2^2 \;\; 1 \quad h_1^2 \;\; + h_1^2 h_2^2 < 1; \tag{11}$$

for $p_{11} + p_{22} > 1$. Therefore, any given parameter con guration could lead to the following three cases: (i) determinacy, when a unique stable MSV solution exists; (ii) indeterminacy, when multiple stable MSV solutions exist; or (iii) explosiveness, when no stable MSV solutions exist. In what follows, we seek to explore the parameter space to identify the regions corresponding to these three cases.[6]

2.3 Determinacy under xed coe cients: Leeper (1991)

It is useful here to recall the necessary and su cient conditions for determinacy of the rational expectations equilibrium (REE) in a xed-coe cient model. Assume thus for the moment that both $_{;t}$ and $_{;t}$ are constant over time and not subject to regime changes, as in Leeper (1991). The log-linearised model is a trivariate dynamic system in the two jump variables $\hat{Y}_t$ and $\hat{}_t$ and the predetermined variable $\hat{b}_t$:

$$\frac{1}{c}\hat{Y}_t = \frac{1}{c}E_t\hat{Y}_{t+1} \qquad \hat{R}_t \quad E_t\hat{}_{t+1} \quad ; \tag{12}$$

$$\hat{}_t = \frac{}{c}\hat{Y}_t + \quad E_t\hat{}_{t+1}; \tag{13}$$

$$\hat{R}_t = \quad \hat{}_t + u_{m;t}; \tag{14}$$

$$\hat{b}_t = \frac{1}{} \quad 1 \quad \frac{}{b} \quad \hat{b}_{t\ 1} \quad \frac{1}{}\hat{}_t + \hat{R}_t \quad \frac{1}{b}u_{\ ;t}; \tag{15}$$

[5] Davig and Leeper (2007) employs a di erent concept of stability, boundedness, which requires bounded paths and thus excludes temporarily explosive paths in one of the two regimes. See Farmer et al. (2009) and Barthelemy and Marx (2015) for a discussion in the context of Markov-switching DSGE models.

[6] Note that the term indeterminacy is used here in a di erent way from that used in the sunspot literature. Since we only consider MSV solutions, we do not consider sunspots in our model. Indeterminacy here means that there is more than one (generally a discrete number of) stable, and thus admissible, MSV solutions.

where c is the steady-state consumption-to-GDP ratio, (1)(1)= determines the slope of the Phillips curve, and hatted variables indicate log-deviations from steady-state values. Using Leeper's (1991) well-known taxonomy, scal policy is said to be passive if the scal rule guarantees debt stabilisation in (15), that is, if the following holds:

$$\frac{1}{\ } \quad 1 \quad \frac{\ }{b} \qquad < 1: \tag{16}$$

In the case of passive scal policy, the following conditions have to hold to yield determinacy:

$$> 1 \quad \text{and} \quad > \frac{1}{\qquad}: \tag{17}$$

The rst condition is the Taylor principle, and it implies the second, which then becomes redundant. According to Leeper's (1991) taxonomy, monetary policy is labelled active if it satis es the Taylor principle; otherwise, it is labelled passive. Hence, the famous result in Leeper (1991) follows: when scal policy is passive, monetary policy needs to be active (i.e., > 1) to yield determinacy.

Conversely, in the case of active scal policy (i.e., when (16) does not hold), monetary policy should be passive to guarantee determinacy: < 1. In this case, a change in lump-sum taxation has real e ects, and the so-called scal theory of the price level holds.[7] The literature often refers to the AM/PF regime by the term Ricardian, while the term non-Ricardian is used for the PM/AF regime. Only in the former case are there no wealth e ects, and thus no e ects on output and in ation, from a change in lump-sum taxes for a given stream of expenditures, as prescribed by Ricardian equivalence. However, this straightforward one-to-one mapping between the policy mix and the Ricardian terminology is possible only if there are no regime changes (or agents are not aware of them). In what follows, we will de ne the conditions for wealth e ects to arise in a Markov-switching context, and hence, we will label Ricardian dynamics the cases in which changes in lump-sum taxes (and hence the debt level) have no e ects on output and in ation. A Ricardian solution thus will feature no wealth e ects from a change in the path of lump-sum taxes in both regimes.

In summary, in a xed-coe cient model, as in Leeper (1991), the determinacy region is de ned by the following conditions:

1. Active monetary/passive scal (AM/PF): > 1 and $(1 \quad)\frac{b}{\ } < \quad < (1 + \quad)\frac{b}{\ }$;

2. Passive monetary/active scal (PM/AF): < 1 and either $< (1 \quad)\frac{b}{\ }$ or $> (1 + \quad)\frac{b}{\ }$:

[7] See Bhattarai et al. (2014) for a thorough analysis of the dynamics implied by such a parameter con guration.

The REE is indeterminate under the PM/PF configuration and explosive under the AM/AF configuration.

2.4 Determinacy under regime switching

Applying the FRWZ method, the Appendix shows that solutions to the model (1)-(6) need to satisfy the following system of equations for the Markov-switching case:

$$
\begin{aligned}
0 = g_{;1}\; 1 + \;\; _{;1} \;\; p_{11}h_1(1 + \;\; + \;\;) + p_{11}^2\, h\, _1^2 + (1 \;\; p_{11})(1 \;\; p_{22})\;\; h\, _1 h_2 g_{;1} \\
+ (1 \;\; p_{11})\, h_1 g_{;2}\, [p_{11}\, h\, _1 + p_{22}\, h\, _2 \;\; (1 + \;\; + \;\;)]\, ;
\end{aligned} \tag{18}
$$

$$
\begin{aligned}
0 = g_{;2}\; 1 + \;\; _{;2} \;\; p_{22}h_2(1 + \;\; + \;\;) + p_{22}^2\, h\, _2^2 + (1 \;\; p_{11})(1 \;\; p_{22})\;\; h\, _1 h_2 g_{;2} \\
+ (1 \;\; p_{22})\, h_2 g_{;1}\, [p_{11}\, h\, _1 + p_{22}\, h\, _2 \;\; (1 + \;\; + \;\;)]\, ;
\end{aligned} \tag{19}
$$

$$
g_{;1} = \frac{\frac{1}{\;} \;\; 1 \;\; \frac{\;}{b} \;\; _{;1} \;\; h_1}{b \;\; \frac{1}{\;} \;\; _{;1}}; \tag{20}
$$

$$
g_{;2} = \frac{\frac{1}{\;} \;\; 1 \;\; \frac{\;}{b} \;\; _{;2} \;\; h_2}{b \;\; \frac{1}{\;} \;\; _{;2}}; \tag{21}
$$

where $p_{11}; p_{22} \in (0;1)$ and the 4 unknowns are h_1, h_2, $g_{;1}$ and $g_{;2}$. Debt, b_t, is the state variable of the system; h_i is the response of debt to its lag in regime i in (8), and $g_{;i}$ is the response of inflation to the lagged debt in regime i (i.e., the element of $g_{i;b}$ that corresponds to inflation in (9)). Determinacy obtains when a single pair $(h_1; h_2)$ satisfies the MSS conditions (10)-(11). As explained above, a solution is Ricardian if $g_{;i} = 0$, for $i = 1; 2$, because inflation dynamics do not depend on the debt level, while they would in the FTPL case (see Bhattarai et al., 2014).

3 Results

3.1 The long-run Taylor principle and the monetary policy frontier

Davig and Leeper's (2007) long-run Taylor principle indicates the conditions that a switching monetary policy needs to satisfy under the two regimes to yield a unique REE in the Markov-switching framework, when assuming a passive fiscal policy. Figure 1a displays what we label the *monetary policy frontier* (henceforth MPF) because it delimits the combinations of monetary policy rule coefficients ($_{;1}$ and $_{;2}$) under the two regimes that deliver determinate equilibria for given fiscal rule coefficients. The combination of monetary policies between the two regimes above the MPF in the gure delivers a unique REE. In contrast, the others monetary policy mixes admit more than one stable solution. The MPF in Figure 1a assumes an always passive scal policy (i.e., $_{;1} = \ _{;2} = 0{:}2$), and thus, it reproduces, in our framework, the long-run Taylor principle: determinacy can obtain even if monetary policy is deviating from the Taylor principle in one of the two regimes. As highlighted by the hatched area in the gure, however, the monetary policy combination above the MPF admits only timid deviations into passive monetary policy. As in Davig and Leeper (2007), asymmetric mean duration would expand the determinacy region in favour of the more transient regime, such that these deviations could be timid for prolonged periods or substantial for brief periods. The intuition is straightforward: in a regime-switching context, the possibility of switching to an active policy in one regime anchors expectations and allows some exibility in monetary policy to relax the Taylor principle in the other regime. The degree of exibility, however, is limited by the overall combination of monetary policies across the two regimes. Uniqueness is guaranteed if monetary policy is \overall" active across the two regimes, that is, if these deviations are timid such that the monetary policy mix is above the MPF. In this case we name the monetary policy mixoverall AM.[8] Finally, it is worth emphasising that the unique solutions inside the MPF deliver standard New-Keynesian dynamics, in the sense that they are Ricardian solutions, as de ned above.

(a) The monetary policy frontier

(b) The scal policy frontier

Figure 1: The monetary and scal policy frontiers.

Notes: Light blue: unique solution; white: indeterminacy; dark blue: explosiveness.

[8] Unfortunately, given the complexity of the system, a meaningful analytic expression for the MPF is not possible. Nonetheless, we will later provide revealing analytic insights for the case of an absorbing regime.

3.2 Allowing flexibility in fiscal policy: The long-run fiscal principle and the fiscal policy frontier

Our setup allows us to map the insights of the MPF into a specular graph for fiscal policy. That is, we can perform a similar analysis, defining the conditions that fiscal policy needs to satisfy to yield a unique REE, for a given combination of monetary policies. Assume that monetary policy is always active ($\ _{;1} = \ _{;2} = 1{:}5$). This will be the symmetric case with respect to Davig and Leeper (2007), who implicitly consider passive fiscal policy in both regimes. Figure 1b displays what we label the fiscal policy frontier (henceforth FPF) because it shows the combination of fiscal policy rule coefficients ($\ _{;1}$ and $\ _{;2}$) under the two regimes that delivers determinate equilibria for the given monetary rule coefficients. As the figure shows, we have determinacy above the FPF. The logic and intuition is the same as for the case of monetary policy. Determinacy can obtain even if fiscal policy is deviating from a passive policy in one of the two regimes, as highlighted by the hatched area in Figure 1b. The possibility of switching to a passive policy in one regime anchors expectations and it allows some flexibility for fiscal policy to be active in the other regime. Uniqueness is guaranteed if fiscal policy is \overall" passive across the two regimes, that is, if these deviations are timid such that the fiscal policy combination is above the FPF. In this case, we name the fiscal policy mixoverall PF.

Given the conditions (18)-(21), the following proposition analytically defines the FPF.

Proposition 1. *The Fiscal Policy Frontier.* For any policy parameter combination, there always exists a particular solution such that in each regime $h_i = \frac{1}{\ }\left(1 - \bar{b}\ _{;i}\right) \equiv h_i(\ _{;i})$ and $g_{;i} = 0$, for $i = 1;2$. This solution thus depends only on $\ _{;i}$ for $i = 1;2$. Then:

(i) For this solution to be MSS, it must be true that h_1 and h_2 satisfy

$$p_{11}\, h_1(\ _{;1})^2 \left\{1 - h_2(\ _{;2})^2\right\} + p_{22}\, h_2(\ _{;2})^2 \left\{1 - h_1(\ _{;1})^2\right\} + h_1(\ _{;1}) h_2(\ _{;2})^2 < 1; \tag{22}$$

which defines the FPF in the space $(\ _{;1};\ _{;2})$.

(ii) The fiscal policy frontier is independent of the monetary policy coefficients.

(iii) This solution yields no wealth effects in both regimes because $g_{;i} = 0$ for $i = 1;2$, and thus, it is a Ricardian solution.

Proposition 1 establishes some important results. First, we can characterise analytically one particular solution. For any fiscal policy mix, i.e., $\ _{;1}$ and $\ _{;2}$, there always exist values $h_1(\ _{;1})$ and

$h_2(\ _{;2})$ such that $g_{;1} = g_{;1} = 0$ in (20) and (21). Hence, these values $h_1(\ _{;1})$ and $h_2(\ _{;2})$ define a solution because they also satisfy (18) and (19). Second, this solution is MSS if it satisfies (22), which defines the FPF in Figure 1b. Third, this is the Ricardian solution: since $g_{;1} = g_{;2} = 0$, inflation and output do not react to a change in the debt level, and thus this solution yields no wealth effects under either regime. Finally, note that both the solution and its stability condition do not depend on the monetary policy coefficients. However, the monetary policy coefficients determine whether other possible MSS solutions exist (see Section 3.3 below).

Corollary 1. *The long-run fiscal principle.* Assume that monetary policy is always active; then, (22) defines the long-run fiscal principle, that is, if the fiscal policy mix $(\ _{;1};\ _{;2})$ satisfies (22), there is a unique Ricardian solution.

The corollary fixes two important results that stem from our analysis. Consider the fiscal stance underlying the long-run Taylor principle in Davig and Leeper (2007) and Figure 1a. It entails an always-passive fiscal policy: the central bank can stabilise the economy by following the Taylor principle or deviating from it, substantially for brief periods or timidly for longer periods, provided that it is backed by a government that implements the fiscal adjustments necessary to stabilise debt. Symmetrically, Figure 1b shows what we can analogously name the *long-run fiscal principle*, given by equation (22): fiscal policy can also deviate substantially from passive behaviour for brief periods or timidly for longer periods and still return determinacy, provided that monetary policy is always active.[9]

Second, Proposition 1 proves that the solution is Ricardian (i.e., $g_{;i} = 0$ for $i = 1;2$), and Figure 1b shows that it is unique because no other MSS solutions exist, given that monetary policy is always active. Figure 1 displays the regions of the determinacy space where solutions are Ricardian and establishes similarity between flexibility in monetary policy and that in fiscal policy, as long as one of the two policies is always AM or PF. This tells us that if one of the two policies is always `well-behaved', the other can deviate timidly without changing the qualitative nature of the dynamics, which remain Ricardian in both regimes. In this case, the fiscal rule guarantees debt stabilisation, and monetary policy controls inflation in both regimes. Inflation is monetary determined and the standard New Keynesian (AM/PF) solution applies in both regimes.

However, our framework admits non-Ricardian solutions when *both* regimes admit a stable solution with wealth effects, i.e., such that $g_{;i} \neq 0$. These solutions would imply spillovers across regimes and FTPL dynamics, as there is no fiscal backing for the government budget constraint. The price level has to move to stabilise debt: inflation is fiscally determined, and monetary policy would not be able

[9] See Section 4 for an example of determinacy after a substantial variation for a brief period.

to fully control inflation. Depending on parameter configurations, these two types of solutions could co-exist, thus leading to multiple stable solutions, as in the white region in Figure 1a. What then are the limits to flexibility to guarantee a unique determinate equilibrium? What are the conditions for wealth effects and FTPL dynamics to arise?

3.3 The extent of flexibility

Thus far, in defining the FPF, we assumed that monetary policy was always active in both regimes. However, how should fiscal policy behave to yield determinacy when monetary policy switches from active to passive in one regime? To answer this question, we need to distinguish two cases, according to whether $_{;2}$ deviates from the Taylor principle to a lesser (case 1) or greater (case 2) extent.

(a) Timid deviations

(b) Substantial deviations

Figure 2: The fiscal policy frontier for different monetary regimes.
Notes: Light blue: unique solution; white: indeterminacy; dark blue: explosiveness.

Case 1: A timid $_{;2}$ deviation. Assume that monetary policy is active under the first regime and deviates only timidly under the second regime ($_{;1} = 1:5$; $_{;2} = 0:97$). In terms of Figure 1a, the long-run Taylor principle is satisfied: the monetary policy mix is above the MPF and overall active. Then, Figure 2a shows that the FPF is the same as in Figure 1a above: again determinacy is preserved if fiscal policy deviates only timidly into the AF territory in one of the regimes. Both monetary and fiscal policy could admit some flexibility as long as the deviations from the AM and PF regime are both timid, such that we have an overall AM combined with an overall PF, which returns what we name an overall AM/PF mix. Both the long-run Taylor principle and the long-run fiscal principle are satisfied. The deviations of both monetary and fiscal policy are so timid that only the Ricardian solution is allowed.

Our analysis suggests that the long-run Taylor principle holds, as long as the long-run fiscal principle does. They are intimately intertwined. Consider a policy mix that lies above the FPF with passive fiscal policy under regime 1 and a timidly active fiscal policy under regime 2. The corresponding MPF for this overall PF is very similar to that in Figure 1a. [10] In other words, as the FPF is unaffected as long as the monetary policy stance is overall active, i.e., the long-run Taylor principle is satisfied, the MPF is also largely unaffected as long as the fiscal stance is overall passive, i.e., as long as the long-run fiscal principle is satisfied. It follows that the long-run Taylor principle ensures determinacy not only when fiscal policy is always passive, as Davig and Leeper (2007) maintain, but also when it deviates timidly into active fiscal territory for some time, provided that the long-run fiscal principle is satisfied.

Case 2: A substantial $_{;2}$ deviation. Now assume a substantial deviation of monetary policy into the passive territory (e.g., $_{;2} = 0{:}9$, see Figure 2b). The monetary policy stance is definitely switching from active to passive, and thus, the long-run Taylor principle is not satisfied and the monetary policy mix cannot be defined overall AM. Instead, we label this mix as the *switching monetary policy*. If the fiscal policy mix remains overall PF, there would be indeterminacy. The long-run Taylor principle is not satisfied, and, in turn, both an always-passive fiscal policy mix (see Figure 1a) or a timid deviation (see Figure 2b) from passive fiscal policy return indeterminacy. Determinacy generally requires fiscal policy to deviate substantially from passive behaviour, too. The fiscal policy mix needs to be below the FPF to yield determinacy, that is, the long-run fiscal principle should not be satisfied. Fiscal policy should deviate from overall PF, definitely switching from passive to active. We name such a fiscal policy mix the *switching fiscal policy*. Note that this is merely a sufficient but not a necessary condition because switching fiscal policies could also return instability, as visible in Figure 2b.

To gain further insight into this result, note that in Figure 2b a new condition appears, represented as a straight line in the space ($_{;1}$; $_{;2}$). This line indicates the threshold for the existence of an MSS non-Ricardian solution: above the line, the policy parameters (and the probabilities of switching) are such that at least one non-Ricardian MSS solution exists, while below the line no stable solution exists. Moreover, we also know from Proposition 1 that a Ricardian MSS solution always exists above the

[10] For the sake of brevity, we omit that figure, which is available from the authors upon request.

FPF. In Figure 2b, the threshold line is below the FPF.[11] Thus, there are at least two stable solutions (one Ricardian and at least one non-Ricardian) for all the fiscal combinations above the FPF, which are therefore indeterminate. Below the line, no stable solution exists. Between the FPF and the line, we find determinacy.[12] There is a unique non-Ricardian solution: Ricardian equivalence does not hold and the dynamics would imply wealth effects in both regimes, meaning that changes in the debt level affect output and inflation. Hence, in the case of switching policies, the unique equilibrium delivers very different dynamics with respect to those defined by the overall AM/PF mix .

3.3.1 An analytical definition of the extent of flexibility: timid vs substantial deviations in the absorbing AM/PF case

We now refer to the case in which regime 1 is AM/PF and absorbing, and hence $p_{11} = 1$. This simplification allows us to derive analytical results on determinacy and, in turn, to develop intuition concerning the numerical results in the general case. We refer the interested reader to the Appendix for the full derivations of the analytical results for the absorbing case. If $p_{11} = 1$, equations (18) and (20) reduce to

$$0 = \frac{\frac{1}{\ } \; 1 \quad \frac{\ }{b} \; _{;1} \quad h_1}{b \; \frac{1}{\ } \quad _{;1}} \; 1 + \quad _{;1} \quad h_1 (1 + \quad + \quad) + \; h_1^2 \qquad (23)$$

and the conditions for MSS, i.e., (10) and (11), simplify to $jh_1j < 1$ and $jh_2j < \frac{1}{\sqrt{p_{22}}}$.

If the economy is already in the absorbing regime 1, the conditions for determinacy are clearly the same as under fixed coefficients. Hence, if fiscal policy is passive, condition (16) must hold ($\frac{b}{\ }(1 \quad) < \quad _{;1} < \frac{b}{\ }(1+ \quad)$),[13] and monetary policy must be active ($_{;1} > 1$). Conversely, the Markov-switching nature of the economy obviously affects the stability condition in the non-absorbing regime. With respect to the fixed-coefficient case, the stability condition (i.e., $jh_2j < \frac{1}{\sqrt{p_{22}}}$) is less binding, the lower the probability of remaining in the second state is. Figure 3a depicts the combinations of the monetary ($_{;2}$) and the fiscal ($_{;2}$) coefficients for the second regime (setting $p_{22} = 0{:}95$) that return determinacy of the Markov-switching equilibrium given an absorbing AM/PF regime 1 ($_{;1} = 1{:}5$;

[11] To be clear, the line lies below the FPF and is not tangent to the FPF, as it might appear from the figure.

[12] The Appendix contains the full analytical characterisation of and an analytical expression for the threshold condition that defines the line in the absorbing case. Such an expression is not available in any meaningful sense for the general case. In general, the slope and position of the line depend on the monetary policy coefficients $_{;i}$ and the switching probabilities p_{ii} : For the parameter combinations in Figure 2b, the line lies below the FPF. For larger switches into PM (i.e., lower $_{;2}$) or different regime persistences, the line could also intersect the FPF. However, our general message remains valid because there will always be a policy combination that yields a unique determinate solution. This again simply reflects the fact that the monetary frontier generally depends on the fiscal policy mix in the two regimes.

[13] Our calibration yields $0{:}019 < \quad _{;1} < 3{:}892$. The calibration is described in Table 1 in the Appendix. We do not discuss it in the main text because it is very standard, and our model is too stylised to make the case for a quantitative analysis. However, the logic of our analyses and results does not depend on the particular calibration chosen.

${}_{;1} = 0{:}2$). Notably, there are two regions in the (${}_{;2}$; ${}_{;2}$) space that return determinacy: an upper-right zone and a lower-left zone.

(a) Absorbing case (b) Non-absorbing case

Figure 3: Determinacy regions when regime 1 is AM/PF.
Notes: Light blue: unique solution; white: indeterminacy; dark blue: explosiveness. The solid lines with arrows indicate a timid deviation from active monetary policy; the dashed lines with arrows indicate a timid deviation from passive scal policy.

First, let us analyse the upper-right zone. In this case, there is MSS if the following conditions concerning regime 2 hold:

$$ {}_{;2} \; 2 \quad {}_{;2}; \frac{b}{} \quad 1 + \sqrt{p_{22}} \qquad ; \qquad (24) $$

$$ {}_{;2} > \quad {}_{;2}; \qquad (25) $$

where ${}_{;2}$ $\frac{b}{}$ 1 $\sqrt{p_{22}}$; and ${}_{;2}$ $\sqrt{p_{22}}$ $\frac{(1 \; \sqrt{p_{22}})(1 \; \sqrt{p_{22}})}{}$: Determinacy clearly emerges when the second regime is AM/PF, too. However, the threshold values ${}_{;2}$ and ${}_{;2}$ imply that both intervals for ${}_{;2}$ and ${}_{;2}$ widen, relative to the xed-coe cients result: there is determinacy even if the second regime deviates from Leeper's (1991) de nition of the AM/PF mix. Careful consideration of the above conditions reveals that ${}_{;2}$ is negative, if $\sqrt{p_{22}} <$, while ${}_{;2}$ is lower than one because $\sqrt{p_{22}} < 1$. In other words, to have determinacy, scal and monetary policy in the second regime are not constrained to always be passive and active, respectively. Rather, they can be \timidly" active and passive, respectively. This e ect is more pronounced the lower p_{22} is. The timid changes for scal and for monetary policy are given by the following intervals (visualised by the dotted and solid arrows in Figure 3a):

$$ {}_{;2} < \ {}_{;2} < \frac{b}{}(1 \quad); \tag{26}$$

$$ {}_{;2} < \ {}_{;2} < 1: \tag{27}$$

Consider now what happens in the lower-left zone. There is global MSS if the following holds:

$$ {}_{;2} < \ {}_{;2} \text{ and } \ {}_{;2} > \frac{b}{} \ 1 + \sqrt{p_{22}} \quad ; \tag{28}$$

$$ {}_{;2} < \ {}_{;2}: \tag{29}$$

In this case, to have determinacy, scal and monetary policies in the second regime are constrained to always be \more than" active and \more than" passive, respectively, relative to Leeper's (1991) conditions. Hence, both monetary policy and scal policy in regime 2 must deviate \substantially" from the other AM/PF regime.

The absorbing case usefully provides easy intuition about the results in the previous sections of the paper, regarding exibility, its limit and the link between the determinacy analysis and the nature of the solutions. First, the MSS condition $|h_2| < \frac{1}{\sqrt{p_{22}}}$ determines the extent of exibility in policies, that is, the admissible \timid" and \substantial" deviations that permit the relaxation of Leeper's (1991) original conditions. In this respect, the persistence of the regime (i.e. p_{22}) plays a key role, as evident from the de nitions of $_{;2}$ and $_{;2}$.

Second, as the absorbing regime is AM/PF, given (23), we know from Leeper (1991) that the only stable solution for this regime implies $g_{;1} = 0$.[14] As for the xed-coe cient New Keynesian model, this solution for the AM/PF regime has no wealth e ects. Then, given (19) and (21) it is easy to show that the upper-right zone in Figure 3a admits only one stable Ricardian solution for the second regime (i.e. $g_{;2} = 0$), while the lower-left zone admits only one stable non-Ricardian/FTPL solution for the second regime (i.e. $g_{;2} \neq 0$). It follows that in the upper-right zone the only stable solution $(h_1; h_2)$ for the whole Markov-switching model implies Ricardian dynamics in both regimes, while in the lower-left zone it implies FTPL dynamics in the second regime.

Figure 3b shows that the general case in which both regimes are non-absorbing ($p_{11}; p_{22} < 1$) exhibits the same qualitative results.[15] In particular, it remains true that the unique stable solution

[14] As the value of $_{;1}$ is greater than 1, no value of $h_1 < 1$ exists, such that the square bracket in (23) is equal to zero.

[15] As Appendix A5 shows, in the general non-absorbing case with our calibration, the threshold values for the scal policy coe cient are $0{:}02 < \ _{;2} < 3{:}93$.

in the upper-right zone is the Ricardian solution, and hence, the dynamics will show no wealth effects in either regime. In contrast, the unique stable solution in the lower-left zone is the non-Ricardian one, and hence, the dynamics of the system will have wealth effects in both regimes.

3.4 The need for coordination within and across regimes: a new taxonomy

The general message from this analysis is that when monetary policy varies timidly, determinacy of the global equilibrium requires that fiscal policy also varies timidly. By contrast, when monetary policy varies substantially, determinacy generally requires fiscal policy to also vary substantially. In a Markov-switching context, monetary and fiscal policies need to coordinate not only within regimes, as suggested by Leeper (1991), but even across regimes. We find that policies need to be overall balanced to guarantee the existence of a unique stable equilibrium. The analysis thus naturally provides a new taxonomy to define both the number and the types of equilibria that could arise when monetary and fiscal policies interact in a Markov-switching context. Two different scenarios are of particular interest. First, overall active monetary policies require overall passive fiscal policies to yield determinacy. This overall AM/PF mix allows a limited degree of flexibility for both monetary and fiscal policy and generates a unique Ricardian solution, with no wealth effects in either of the two regimes. Second, switching monetary policies must be paired with switching fiscal policies to yield determinacy. This overall switching mix generates a unique non-Ricardian solution, which implies wealth effects and FTPL dynamics in both regimes.

3.4.1 The importance of coordination across regimes

Leeper (1991) is the seminal paper on the importance of coordination between monetary and fiscal policies. However, his taxonomy is defined for a fixed-coefficient model and does not provide any guidance in a Markov-switching context. In such a framework, coordination is not merely a question of being active or passive, but the extent to which policies are active or passive across regimes is essential. Moreover, the expectation of a stable regime in the future is not per se sufficient to achieve determinacy, or nothing ensures that switching between two regimes, which are determinate in a fixed-coefficients context, yields determinacy. Similar to the Leeper's (1991) approach in the case of fixed coefficients, our proposed taxonomy provides conditions for determinacy, but we also provide conditions for the presence (or absence) of wealth effects in both regimes.

To make this point clear, consider again Figure 3b. Point B in the upper-right zone and point A on its left return determinacy and indeterminacy, respectively. This is true even if both points exhibit the same fiscal policy in both regimes (and the same monetary policy in regime 1) and correspond to an economy that switches between an AM/PF and a PM/AF mix: two regimes that, taken in isolation, are determinate.[16] The same result obtains if one compares point B_1 in the lower-left zone and point A, characterised by the same monetary policy in both regimes (and the same fiscal policy in regime 1). To explain these apparently puzzling findings, we exploit our taxonomy and the MPF and FPF concepts. Points A and B entail the same timid deviation in regime 2 from the passive fiscal policy under regime 1, and thus, they are both characterised by an overall PF. To have determinacy of the global equilibrium, monetary policy should also vary timidly in regime 2, to have an overall AM. This does not happen at point A, as monetary policy is insufficiently active, while it does at point B (which indeed lies in the determinate area in Figure 2a). Point B is above both the MPF and the FPF, that is, it satisfies both the long-run Taylor principle and the long-run fiscal principle. Point A satisfies only the latter. Compare now points A and B_1. As these two points share the same substantial deviation in regime 2 from the active monetary policy under regime 1, they do not satisfy the long-run Taylor principle, yielding a switching monetary policy. To have determinacy of the Markov-switching system, fiscal policy is also required to switch, i.e., to vary substantially. This is not the case for point A, which lies above the FPF, as fiscal policy is only timidly active, and thus, fiscal policy is overall PF. In contrast, this is the case for point B_1, which is below the FPF (and thus lies in the determinate area in Figure 2b).

Furthermore, determinacy could arise from very different policy mixes. Switching from a double active regime (AM/AF, explosive in fixed coefficients) to a double passive one (PM/PF, indeterminate in fixed coefficients) can return determinacy. Consider, for example, point C in Figure 2a and point D in Figure 2b. They share the same fiscal policy coefficients, satisfying the long-run fiscal principle above the FPF: a passive fiscal policy under regime 2 and a timid deviation from it under regime 1 (i.e., an overall PF). In both cases, monetary policy is active in the first regime and passive in the second. In Leeper's (1991) taxonomy, this would be a shift from a double active to a double passive regime that returns determinacy in Figure 2a but not in Figure 2b. According to our interpretation, this is because at point C the overall monetary and fiscal policy mix is balanced (i.e, there is also a timid change in monetary policy that satisfies the long-run Taylor principle, and thus, we are in the case of overall AM/PF), while at point D it is not (there is a substantial change in monetary policy, which is switching).

[16] The coordinates of the points in Figure 3b are A: ($_{;2}$ = 0:9; $_{;2}$ = 0); B: ($_{;2}$ = 0:97; $_{;2}$ = 0); B_1: ($_{;2}$ = 0:9; $_{;2}$ = 0:05).

A second important aspect of the importance of coordination across regimes regards the validity of the long-run Taylor principle disclosed by Davig and Leeper (2007). Bringing scal policy into the picture shows that the long-run Taylor principle is conditional on scal policy behaviour: it holds only if the scal policy mix is overall PF. The long-run Taylor principle fails when scal policy is not overall passive, as, for example, at a point in the dark blue region of Figure 2a.

3.5 The expectation e ects of regime shifts

This section considers, in greater detail, the dynamics implied by the di erent solutions to illustrate the link with our proposed taxonomy, the determinacy analysis and the role of \expectation e ects". Cross-regime spillovers characterise the dynamics in a Markov-switching framework, as the economy's equilibrium properties are contaminated by both the characteristics of the other regimes and the probability of shifting between the alternative regimes. Davig and Leeper (2008) de ne \expectation e ects" as the di erence between the equilibrium outcomes of a model with xed coe cients and those of a model that accounts for expected changes in regimes.

Recall that we identi ed two sets of determinate parametrizations: an overall AM/PF mix that yields Ricardian dynamics in both regimes and an overall switching mix that implies wealth e ects in both regimes. Points B and B_1 in Figure 3b are examples of these two types of solutions. Both are characterised by a shift from the same AM/PF regime to a PM/AF regime with transition probabilities $p_{11} = p_{22} = 0{:}95$, and they both return determinacy. While point B entails a timid deviation of both monetary and scal policy from the AM/PF regime, for point B_1 the deviation is substantial. As a consequence, point B is an overall AM/PF mix (see Figure 2a), and point B_1 is an overall switching mix (see Figure 2b). Figure 4 shows the impulse responses to a positive scal shock (i.e., an unexpected reduction in lump-sum taxes) for the policy combinations implied by points B and B_1 (panels a and b, respectively).[17] The impulse response functions are conditional on remaining in the particular policy regime in place, and thus, each panel displays two columns of graphs corresponding to each of the two regimes. Moreover, to highlight the expectation e ects, each panel in Figure 4 displays two lines: the dashed lines are the responses of the variables under a xed-coe cients model, while the solid lines are the responses under a Markov-switching model. The di erence between the solid and the dashed lines in each graph represents the expectation e ects.

For the overall AM/PF mix (i.e., point B) we have the following:

[17] Recall that the policy combinations are for point B and regime 1 ($_{;1}$ = 1:5; $_{;1}$ = 0:2) and for regime 2 ($_{;2}$ = 0:97; $_{;2}$ = 0); for point B_1, regime 1, we have ($_{;1}$ = 1:5; $_{;1}$ = 0:2) and for regime 2, we have ($_{;2}$ = 0:9; $_{;2}$ = 0:05).

1. The solid lines across the two regimes in Figure 4a are coincident except for the path of debt. The possibility of moving towards a regime with Ricardian dynamics makes the impulse responses also behave as Ricardian in the PM/AF regime (i.e., in ation does not increase).

2. Now consider the di erences between the solid and dashed lines. The expectation e ects are asymmetric in the two regimes. In the AM/PF regime, the expectations e ects are absent, as there is no di erence between these two lines.

Regarding the overall switching mix (i.e., point B_1) we have the following:

1. The solid lines no longer coincide. In contrast to the previous case, the possibility of switching to a PM/AF regime makes the impulse responses non-Ricardian also in the AM/PF regime: wealth e ects are at work in both regimes, and in ation now increases under both regimes.

2. The expectation e ects are again asymmetric, and there are now wealth e ects under the AM/PF regime.

(a) overall AM/PF regime

(b) overall switching regime

Figure 4: Impulse response function to a positive scal shock.

Notes: Regimes are recurrent, with $p_{11} = p_{22} = 0{:}95$. Blue solid lines: Markov-switching model; red dashed lines: xed coe cients model.

Why do impulse responses for these two points, which entail a switch from an AM/PF to a PM/AF regime, return such strikingly different results?[18] Our analysis above explains the underlying mechanisms that drive these findings. We labelled an overall AM/PF mix one in which only timid deviations from AM/PF are allowed. In this case, there is only one type of admissible stable solutions: those above the FPF in Figure 2a. However, as implied by Proposition 1, we know that these solutions yield Ricardian dynamics. There are no wealth effects under the AM/PF regime, and there are strong inflation-anchoring expectation effects under the PM/AF regime. The possibility of switching to the AM/PF regime, once in a PM/AF regime, is here sufficient to stabilise inflation under both regimes. The cross-regime spillover of the AM/PF regime dominates and its dynamics pass through to the PM/AF regime. As a result, the two regimes behave identically, except for the path of debt, because conditional on the PM/AF regime, taxes do not increase to stabilise the debt. Conversely, for an overall switching mix, Figure 2b shows that determinacy requires both policies to substantially switch across regimes, and the unique stable solutions in this case exhibit non-Ricardian dynamics under both regimes. The possibility of switching to the AM/PF regime is in this case not sufficient to stabilise inflation under both regimes. The cross-regime spillover of the PM/AF regime dominates, and its dynamics pass through to the AM/PF regime. Thus, there are wealth effects in both regimes: inflation increases under both regimes, and fiscal policy thus undermines the ability of monetary policy to control inflation. Inflation increases less under the AM/PF regime due to the reaction of monetary policy. In this case, the expansionary fiscal policy steps on a rake, as monetary policy causes a through its attempt to control inflation.

Finally, do wealth effects disappear if agents are confident in a once-and-for-all switch to an AM/PF regime? We find that under an overall AM/PF mix, the impulse responses are identical to those in Figure 4a even if the AM/PF regime is absorbing, i.e. $p_{11} = 1$. Conversely, in the overall switching mix case with an absorbing AM/PF regime, the impulse responses in the PM/AF regime do not differ from those in the non-absorbing case. Our new taxonomy explains that the expectation of an absorbing AM/PF regime for the future is neither a necessary nor a sufficient condition to avoid wealth effects in the PM/AF regime. It is not necessary because we do not find wealth effects in the overall AM/PF case even when $p_{11} = p_{22} = 0.95$ (see Figure 4a). It is not sufficient because we find wealth effects in the PM/AF regime in the overall switching case even when the AM/PF regime is absorbing.

[18] Both the results of asymmetric expectation effects and the coincidence of the solid lines under point B but not under point B_1 hold even when considering a monetary policy shock. These results are available from the authors upon request.

4 Some theoretical and policy implications

Our framework and methodology has several implications, and it can rationalise some apparently contradictory results in the literature.

First, our new taxonomy provides an answer to the problem of establishing whether a regime is Ricardian in a model in which agents are aware of recurrent regime changes. In a Markov-switching context, as Bianchi and Melosi (2013) note, the policy mix according to Leeper's (1991) taxonomy is insu cient to establish whether a regime is Ricardian. However, we nd that neither expectation e ects nor wealth e ects are present under an AM/PF regime when agents expect a regime shift and the policy mix is overall AM/PF . Even more so, anoverall AM/PF mix is de nitively Ricardian in both regimes AM/PF and PM/AF.

Second, the overall AM/PF mix is consistent with the case advanced by Krugman (2014) of a \timidity trap". Take an unbacked scal expansion engineered to escape a liquidity trap. If that PM/AF policy deviates only timidly from the previous AM/PF regime, that is, if the policy action is too timid, it would not bring about the wealth e ects needed to re ate the economy. To have the desired e ects, there should be a clear departure from the previous regime, hence an overall switching mix. Section 4.1 shows that this insight is particularly relevant for the recent zero lower bound episode.

Third, our results are consistent with Liu et al. (2009) who analyse regime shifts in monetary policy in a context of an always-passive scal policy and nd that the expectation e ects are asymmetric.[19] The shift from a dovish (or less hawkish) monetary regime to a hawkish one reduces in ation volatility to a greater extent than an inverse shift raises it: in ation-anchoring expectations prevail.

Our methodology does not replicate the ndings in Chung et al. (2007) that scal theory is always at work when agents assign a positive probability of moving towards active scal policy (e.g., point B). This is because our method and stability concept are di erent. MSS stability accepts as equilibrium all paths with stationary rst and second moments, taking into account the possibility of regime changes. We believe that this is a highly relevant concept because it is coherent with the standard transversality condition that requires equilibrium paths to be bounded in expectation. However, it follows that MSS does not impose stationarity of all regimes taken in isolation, and hence, it does not exclude temporary explosive dynamics. Nonetheless, agents' expectations are still nite at every horizon, as agents take into account the possibility of regime changes in forming their forecasts. As such, MSS could rationalise episodes such as hyperin ation or bubble boom-and-bust dynamics. In our case, MSS

[19] This result holds even if the two regimes have the same transition probabilities.

admits as equilibrium a temporary explosive path for the level of debt as in Figure 4a. Conditional on staying in regime 2 the debt level would explode, but the probability of staying in a given regime for long periods tends quickly to zero. A stationary solution would exist if temporary explosive dynamics were not too persistent. Chung et al. (2007) employ bounded stability, an alternative stability concept that instead excludes temporarily explosive paths.[20] Following Foerster (2016), to highlight how our results change when such equilibrium paths are not considered, we use a more tractable but related concept: "both regime stable" (BRS), where the only admissible equilibria imply $h_i < 1$ for $i = 1;2$.[21] h_i would generally depend on all the policy coefficients and the probabilities of switching. However, the Ricardian solution in Proposition 1 is defined by $h_i(\ _{;i})$ and thus depends only on fiscal policy coefficients, not on both the probabilities of switching and the monetary policy coefficients. Hence, the BRS condition $h_i < 1$ simply coincides with the standard passive fiscal policy condition in each single regime i, rather than with the MSS condition (22). It follows that BRS excludes any flexibility in fiscal policy and the FPF will be defined by Leeper's (1991) passive fiscal policy conditions, as usual. Point B would not be within the FPF because it implies paths with explosive debt levels, despite that being an extremely unlikely event.[22]

Furthermore, our paper is consistent with the results in Bianchi and Melosi (2013) who find that after a deficit shock under an AM/PF regime or under a short-lasting deviation towards a PM/AF regime, there are no effects on inflation (or output). Inflation and output, however, increase under a long-lasting deviation towards the same PM/AF. As both the short- and the long-lasting deviations are towards the same PM/AF, the authors identify regime persistence as the key determinant to establish whether a regime is Ricardian.[23] Our taxonomy is consistent with this finding because, although we have not focused on the role of transition probabilities thus far, our definitions of timid deviation and overall mix depend on them (see Section 3.3.1). Consider a numerical example in the case of the overall switching mix described by point B_1, reported as a black dot in Figure 5. Under this policy combination, if the second regime is long-lasting, say $p_{22} = 0.95$, a timid deviation is defined by $\ _{;2} \in [\ 0.021; 0.02]$ and $\ _{;2} \in [0.955; 1]$. Instead, if regime 2 is less persistent, say $p_{22} = 0.8$, a timid deviation is defined by $\ _{;2} \in [\ 0.16; 0.02]$ and $\ _{;2} \in [0.67; 1]$.[24] Therefore, with a long-

[20] See their analytical example.

[21] See the discussion on p. 217 in Foerster (2016). Note that BRS is a necessary, but not sufficient condition for bounded stability.

[22] See the Appendix for a representation of how the monetary and fiscal frontiers change when the BRS stability concept is employed.

[23] See also Bianchi and Ilut (2014) on this point. They do not find any effect of a tax shock on inflation when the AM/PF regime is perceived to be fully credible (if agents expect to remain there forever) or if, being in a PM/AF regime, agents are confident in a return to the AM/PF regime.

[24] These intervals can be obtained following the procedure in Section 3.3.1 and Appendix A5.

lasting deviation, B_1 would correspond to an overall switching mix (see Figure 5a), while with a less permanent deviation we would have an overall AM/PF regime (see Figure 5b). In the first case, the impulse responses to a fiscal shock would display a hike in inflation (see Figure 6a) because the unique stable solution is the non-Ricardian one. In the second, there would not be inflationary effects (see Figure 6b) because the unique stable solution is the Ricardian one.

(a) $p_{22} = 0.95$

(b) $p_{22} = 0.80$

Figure 5: The monetary policy frontier for different levels of persistence of regime 2.
Notes: Light blue: unique solution; white: indeterminacy; dark blue: explosiveness.

This analysis could have notable consequences for monetary policy: for both the timing of any exit strategy and forward guidance. During the recent crisis, the accumulated credibility of the Federal Reserve permitted well-anchored inflation expectations, despite that the U.S. was potentially in a PM/AF regime. If we are prepared to believe that during the crisis monetary policy deviated substantially from an AM regime, then the only way to avoid a future spike in inflation is to make this deviation short-lasting. A long-lasting deviation, conversely, could either de-anchor inflation expectations and make inflation unavoidable or generate multiple solutions, depending on the behaviour of fiscal policy. Indeed, if fiscal policy remains only timidly active, it may be difficult for policy makers to predict an inflationary surge, as both the Ricardian and the non-Ricardian solutions are admissible. An inflationary surge would be possible in the latter case. If we believe this scenario to be the relevant one, then it might be that the observed subdued path of inflation is due to agents coordinating on a Ricardian solution. However, these dynamics could abruptly revert into an inflation upswing if

(a) $p_{22} = 0{:}95$ (b) $p_{22} = 0{:}80$

Figure 6: Impulse response function to a positive fiscal shock for different levels of persistence of regime 2.
Notes: Blue solid lines: Markov-switching model; red dashed lines: fixed coefficients model.

expectations about the behaviour of fiscal policy were to suddenly switch.

4.1 An application to the ZLB

During the Great Recession, the Federal Reserve, the European Central Bank, the Bank of England and the Bank of Japan all brought their policy rates towards the ZLB with the aim of stimulating economic activity. This is a very special case of passive monetary policy, which can be described in our model by assuming = 0, i.e., the central bank avoids moving interest rates as inflation changes. In this section we want to conduct two exercises relative to the ZLB regime. First, imagine being in a crisis regime with interest rates stuck at the ZLB and agents expecting a return to business-as-usual regime|the traditional AM/PF regime. Then, how should fiscal policy be fixed in the current regime to guarantee the existence of a unique equilibrium? Which type of solution would that be? How long should the ZLB regime be in place? Figure 7 depicts determinacy results for different fiscal coefficients ($_{;2}$) and durations (p_{22}) of crisis regime 2, when $_{;2} = 0$ and an AM/PF regime 1 is expected with probability $p_{11} = 0{:}95$.

Figure 7: Determinacy and ZLB.

Notes: Light blue: unique solution; white: indeterminacy.

The rst clear-cut result is that if the ZLB regime 2 is short-lasting (p_{22} 1), then there is indeterminacy, whichever scal policy is adopted. Second, determinacy is unattainable when scal policy is passive, as $_{;2}$ should be below the dotted line that represents the cuto value for passive scal policy. Third, the upper bound of the determinacy area is an upward-sloping line: the more short-lived the ZLB is, the more active scal policy must be. To have determinacy, agents must expect the ZLB to last for a long period of time and to be accompanied by an active scal policy. In the event that there were such a unique stable solution, it would be the result of an overall switching policy mix (a switching monetary policy combined with a switching scal policy), and hence, expectation e ects would be operative. As in Figure 4b, the FTPL would apply, and an unbacked scal expansion would spur output, increase in ation and lower real debt: all desirable outcomes in a period of economic stagnation, below-target in ation and high indebtedness.

Two important points stem from these results. The rst is the inadequacy of the conventional New Keynesian model, which does not consider active scal policies. The other is the importance of \forward guidance", both on the monetary and on the scal side. Even if agents expect to return in the future to the AM/PF regime, the question of how long the present policy is expected to last is key. To obtain determinacy, agents must be convinced of a long-lasting deviation from the AM/PF regime both through the promise, on the monetary side, of a long period of zero interest rates and, on the scal side, of a long period of no tax increases or spending cuts.

Conversely, in the presence of a passive fiscal policy, now and in the future, or of a short-lasting deviation from it, there would be multiple equilibria under a ZLB (the white area in the figure). These consist of two stable solutions: one characterised by Ricardian dynamics and the other by non-Ricardian dynamics. In this case, inflation would become indeterminate. This could be the outcome for those countries, currently at their ZLB, where austerity imposes constraints on fiscal policy (the eurozone) or where fiscal policy is mainly passive (Japan).

Figure 8: Impulse response functions from a BVAR

We now perform a second exercise to empirically investigate the policy mix in place in the U.S. during the ZLB period, through the lenses of our model. We examine the empirical impulse response functions computed using a Bayesian VAR fitted on quarterly data for the sample period 2008q4-2015q4. To assess the effect of a fiscal shock, we consider data for the primary deficit-to-GDP ratio, the federal funds rate, real GDP, the GDP deflator and real debt.[25] During this period, monetary policy can be safely assumed to be passive, while there is more room for debate over the fiscal policy stance. According to our model, we can gain some insights into how agents perceive the prevalent policy mix by comparing the pattern of VAR-based impulse responses following a positive fiscal shock to the corresponding theoretical responses shown in Figure 4.

As Figure 8 shows, the impulse response functions are consistent with the PM/AF case of an overall AM/PF mix: while output and inflation do not move, there is a run-up in real debt. Assuming that agents expect to return to the same AM/PF equilibrium as in Figure 7, the only equilibrium consistent with these dynamics for the real debt is a Ricardian equilibrium due to timid AF policies in the indeterminacy (white) area in Figure 7.[26] Through the lens of our model, therefore, the data for the ZLB period suggest that the U.S. could well be in an indeterminate region and in a \timidity trap", where agents are coordinating on the Ricardian solution. This justifies the observed subdued path of inflation. Note that a more aggressive fiscal policy would eliminate one of the two equilibria and guarantee determinacy. Alternatively, an exogenous event, such as the election of a new government, could switch agents to coordinate on another possible equilibrium in the indeterminacy area, causing a spike in inflation, with the Fed being unable to do anything to avoid it.

5 Conclusions

This paper studies the determinacy properties of the equilibrium in a New Keynesian model when both monetary and fiscal policies may switch according to a Markov process. Nothing ensures that the switching between two regimes, which would be determinate under a fixed-coefficient framework, returns determinacy. Davig and Leeper (2007) define the long-run Taylor principle as the condition that the coefficients in the Markov-switching Taylor rule need to satisfy to guarantee a unique equilibrium, given passive fiscal policy. This can be graphically visualised as a monetary frontier. Equivalently, we define a fiscal frontier that visualises the long-run fiscal principle as the condition that the coefficients in the Markov-switching government tax rule need to satisfy to guarantee a unique equilibrium, given

[25] The primary deficit-to-GDP ratio was constructed from NIPA data using the procedure illustrated in Bianchi and Melosi (2014), while real debt was computed by dividing the Market Value of U.S. Government Debt (FRED code: MVGFD027MNFRBDAL) by the GDP deflator. The remaining three variables were also taken from the FRED database. Real GDP, real debt and the GDP deflator are considered in log-levels. Our VAR includes two lags and a constant term, and we identify fiscal shocks by means of a recursive scheme in which the deficit-to-GDP ratio is ordered before all other variables. We use a Normal-Wishart prior augmented with two sets of dummy observations, i.e., both sums-of-coefficients and dummy initial observations, to accommodate the presence of unit roots and cointegration in the data (see Sims and Zha, 1998).

[26] Impulse response functions undertaken when employing aggressive fiscal policies (either active or passive) return a decreasing path for the real debt, as in Figure 4b. These results are available from the authors upon request.

active monetary policy. We find that the long-run Taylor principle ensures determinacy not only with an always-passive fiscal policy, as Davig and Leeper (2007) maintain, but also when it deviates timidly into active fiscal territory for some time, provided that the long-run fiscal principle is satisfied.

For the economy to exhibit a unique stable rational expectations equilibrium, monetary and fiscal authorities should coordinate not only within regimes as suggested by Leeper (1991), but also across regimes by choosing the extent of activeness or passiveness. Hence, we propose a new taxonomy that generalises the seminal paper of Leeper (1991) to a Markov-switching context. We name a timid deviation from an active monetary policy into passive monetary territory that respects determinacy—i.e., that satisfies the long-run Taylor principle—an "overall active monetary policy". Symmetrically, a timid deviation from a passive fiscal policy into active fiscal territory—i.e., that satisfies the long-run fiscal principle—is named an "overall passive fiscal policy". Substantial shifts in monetary and fiscal policies are termed "switching policies". Monetary and fiscal policies need to be overall balanced to guarantee a unique equilibrium: overall active monetary policies need to be coupled with overall passive fiscal policies (i.e., an overall AM/PF mix), and switching monetary policies with switching fiscal policies (i.e., an overall switching policy mix).

Our new taxonomy also establishes an explicit link between the determinacy analysis and the dynamic behaviour of a Markov-switching DSGE model. If the policy mix is overall switching, then the fiscal theory of the price level is always at work. This is not true if the policy mix is overall AM/PF. In this latter case, there are no wealth effects because fiscal policy deviates timidly into the active territory remaining, nevertheless, overall passive. As a result, the timid monetary-fiscal regime changes from the AM/PF benchmark, keeping fiscal expectations anchored, allowing the central bank to avoid wealth effects and to keep inflation under control. Moreover, the expectation of a fully credible (even absorbing) AM/PF regime for the future is neither a necessary nor a sufficient condition to avoid wealth effects under a PF/AM regime.

Our framework has a number of policy implications that we discussed in Section 4. In particular, our model suggests that a "timidity trap" and an indeterminate equilibrium can explain the empirical evidence for U.S. data during the crisis. An important implication for the ability of the central bank to control inflation, is that there could be an inflation upswing if the expectations regarding the behaviour of fiscal policy were to suddenly switch in a substantial way.

The analysis suggests some directions for future research. Our results are based on a very simple New Keynesian model. The advantage of such a framework is to allow us to obtain a number of analytical results, to gain insightful intuitions into what drives determinacy and the linkage among determinacy, dynamics, expectation effects and wealth effects. The natural next step in this line of research would be to determine the extent to which our new taxonomy and results help to interpret the numerical results in a more realistic, and possibly estimated, DSGE model. Finally, our definition of timid deviation has the same flavour as Leeper and Zha's (2003) definition of “modest policy interventions.” However, our definition is based on the determinacy region of the parameter space and not on their modesty statistic. Empirically evaluating whether these definitions are consistent could be a fruitful avenue for future research.

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A Appendix

A.1 Parametrization

Table 1. Calibration

Parameter	Value	Description
	0:99	Intertemporal discount factor
	11	Dixit-Stiglitz elasticity of substitution
	0:75	Calvo probability not to optimise prices
N	0:333	Hours worked
b	0:4	Debt-to-GDP ratio
c	0:8	Consumption-to-GDP ratio

A.2 The model

In the paper we use a simple New Keynesian model with scal policy.

Households. The representative household maximises lifetime utility

$$E_0 \sum_{t=0}^{\infty} {}^{t} (\log C_t \quad N_t) \tag{30}$$

under a sequence of budget constraints given by

$$P_t C_t + (1 + i_t)^{1} B_t \quad P_t w_t N_t + F_t \quad {}_t + B_{t \; 1}: \tag{31}$$

E_0 is the expectations operator conditional on time $t = 0$ information, C_t is real consumption, N_t is labour, w_t is the level of real wages, F_t are pro ts, ${}_t$ are taxes, $R_t = 1 + i_t$ is the gross return on bonds purchased at date t (i.e., B_t). Maximization yields the rst order conditions

$$1 = \quad E_t \frac{P_t}{P_{t+1}} (1 + i_t) \frac{C_t}{C_{t+1}} \quad ; \tag{32}$$

$$w_t = \quad C_{\;t}: \tag{33}$$

Final good producers. In each period, a nal good Y_t is produced by perfectly competitive rms, using a continuum of intermediate inputs $Y_{i;t}$ indexed by $i \; 2 \; [0; 1]$ and a standard CES production function $Y_t = \int_0^1 Y_{i;t}^{(\quad 1)=} di^{\;=(\quad 1)}$, with $\; > 1$. Final good producers' demand schedules for intermediate good quantities are $Y_{i;t} = (P_{i;t} = P_t) \quad Y_t$, where the aggregate price index is de ned as

$$P_t = \left(\int_0^1 P_{i;t}^{1} \; di \right)^{1=(1 \quad)}.$$

Intermediate goods producers. There exists a continuum of intermediate goods produced by rms with constant returns to scale production function: $Y_{i;t} = N_{i;t}$. Intermediate goods producers compete monopolistically and set prices according to the usual Calvo mechanism. In each period each rm has a xed probability 1 to re-optimise its nominal price $P_{i;t}$ in order to maximise pro ts. With probability , instead, the rm keeps its nominal price unchanged. Using the stochastic discount factor $Q_{t;t+j} = \; ^j \frac{P_t C_t}{P_{t+j} C_{t+j}}$, the rst order condition of the rm's problem gives the optimal relative price

$$p_{i;t} \quad \frac{P_{i;t}}{P_t} = \frac{}{1} \frac{E_t \sum_{j=0}^{1} (\quad)^j \frac{Y_{t+j}}{C_{t+j}} \frac{P_{t+j}}{P_t} \quad w_{t+j}}{E_t \sum_{j=0}^{1} (\quad)^j \frac{Y_{t+j}}{C_{t+j}} \frac{P_{t+j}}{P_t} \quad ^{1}} : \tag{34}$$

As in Ascari and Ropele (2009), we introduce two auxiliary variables that allow to rewrite the last expression recursively

$$_t \quad E_t \sum_{j=0}^{1} (\quad)^j \frac{Y_{t+j}}{C_{t+j}} \frac{P_{t+j}}{P_t} \quad w_{t+j} = \frac{Y_t}{C_t} w_t + \quad E_t \left[\quad _{t+1} \quad _{t+1} \right] ; \tag{35}$$

$$_t \quad E_t \sum_{j=0}^{1} (\quad)^j \frac{Y_{t+j}}{C_{t+j}} \frac{P_{t+j}}{P_t} \quad ^{1} = \frac{Y_t}{C_t} + \quad E_t \left[\quad _{t+1}^{1} \quad _{t+1} \right] ; \tag{36}$$

where $_t \quad P_t = P_{t-1}$ is the aggregate gross rate of in ation. As all re-optimizing rms face the same problem and pick the same relative price, aggregate in ation evolves according to

$$1 = \quad _t^{\;1} + (1 \quad) \; p_{i;t}^{\;1} \quad : \tag{37}$$

Individual rms demand labour according to the relation $N_{i;t} = (P_{i;t} = P_t) \quad Y_t$. Aggregating this expression yields $N_t = Y_t s_t$, where $N_t \quad \int_0^1 N_{i;t} \, di$ and $s_t \quad \int_0^1 (P_{i;t} = P_t) \quad di$. The variable s_t measures the dispersion of relative prices across intermediate rms. s_t is bounded below at one and it represents the resource costs (or ine ciency loss) due to relative price dispersion under the Calvo mechanism. s_t can be written recursively as

$$s_t = (1 \quad) \; p_{i;t} \quad + \quad (\quad _t) \quad s_{t-1} : \tag{38}$$

Fiscal and monetary policies. The government budget constraint is given by

$$(1 + i_t)^{1} b_t = \frac{b_{t\ 1}}{t} + G \quad {}_t ; \tag{39}$$

where b_t $B_t = P_t$, G, and ${}_t$ are the levels of government debt, expenditure, and taxes, all in real terms. Note that we assumed for simplicity that the government chooses a constant level of expenditure G. Taxes are set according to the scal policy rule

$$_t = \frac{b_{t\ 1}}{b}^{;t} e^{u\ ;t} ; \tag{40}$$

while the central bank sets the interest rate following the simple Taylor rule

$$R_t = R \ {}_t^{;t} \ e^{u_{m;t}} : \tag{41}$$

Note that the parameters ${}_{;t}$ and ${}_{;t}$ are indexed with time as they can take di erent values according to the underlying Markov-switching process.

Complete nonlinear model. After imposing market clearing (with $Y_t = C_t + G$), the dynamics of aggregate variables is described by the following set of equations (here reproduced for convenience)

$$1 = E_t \ \frac{R_t}{t+1} \frac{Y_t \quad G}{Y_{t+1} \quad G}$$

$$w_t = (Y_t \quad G)$$

$$1 = {}_t^{\ 1} + (1 \quad) \ p_{i;t}^{\ 1}$$

$$p_{i;t} = \frac{}{1} \frac{t}{t}$$

$$_t = \frac{Y_t}{Y_t \quad G} w_t + E_t \left[{}_{t+1} \ {}_{t+1} \right]$$

$$_t = \frac{Y_t}{Y_t \quad G} + E_t \left[{}_{t+1}^{\ 1} \ {}_{t+1} \right]$$

$$s_t = (1 \quad) \ p_{i;t} + {}_t s_{t\ 1}$$

$$N_t = s_t Y_t$$

$$\frac{b_t}{R_t} = \frac{b_{t\ 1}}{t} + G \quad {}_t$$

$$_t = \frac{b_{t\ 1}}{b}^{;t} e^{\ u\ ;t}$$

$$R_t = R \ {}_t^{;t} \ e^{m\, u_{m;t}}$$

Zero-in ation steady state. If we switch o the exogenous processes $u_{;t}$ and $u_{m;t}$, we can solve for the zero-in ation steady state

$$
\begin{aligned}
R &= \quad {}^{1} \\
w &= \quad cY \\
p_i &= 1 \\
&= \frac{1}{c(1 \qquad)} w \\
&= \frac{1}{c(1 \qquad)} \\
s &= 1 \\
N &= sY = Y \\
&= (1 \quad c) + b(1 \qquad) \; Y
\end{aligned}
$$

where we used the ratios $c \quad C = Y$ and $b \quad b = Y$. Further, note that the equation

$$
p_{i;t} = \frac{\quad}{1} \frac{t}{t}
$$

implies the following parameter restriction

$$
1 = \frac{\quad}{1} - = \frac{\quad}{1} Y \; c: \tag{42}
$$

Log-linearised model. The nonlinear model can be log-linearised around the non-stochastic zero-in ation steady state. Standard computations lead to

$$
\begin{aligned}
\frac{1}{c}\hat{Y}_t &= \frac{1}{c} E_t \hat{Y}_{t+1} \qquad \hat{R}_t \quad E_t \, \hat{}_{t+1} \quad ; \\
\hat{}_t &= \frac{}{c} \hat{Y}_t + \quad E_t \, \hat{}_{t+1} \, ; \\
\hat{R}_t &= \quad {}_{;t} \, \hat{}_t + \quad {}_m u_{m;t} \\
\hat{b}_t &= \frac{1}{} \quad 1 \quad \frac{}{b} \quad {}_{;t} \quad \hat{b}_{t \quad 1} \quad \frac{1}{} \hat{}_t + \hat{R}_t \quad \frac{1}{} \frac{}{b} \quad u_{;t} \, ;
\end{aligned}
$$

with $\quad (1 \quad)(1 \qquad) = \,$. These equations correspond to equations (12)-(15) in the main text.

A.3 The FRWZ solution method

The analysis of determinacy under Markov-switching coe cients can be performed by checking the existence of a unique stable MSV solution. In order to nd all the MSV solutions, we adopt the perturbation method proposed by FRWZ. The method can be applied directly on the nonlinear version of the model. However, instead of considering the complete nonlinear model outlined above, it is convenient to manipulate the equations and reduce the dimensionality of the system. The smaller system turns out to be:

$$1 = E_t \frac{{}_t^{\;;t} e^{u_{m;t}}}{{}_{t+1}} \frac{Y_t \quad G}{Y_{t+1} \quad G};$$

$$\left(\frac{1 \quad {}_t}{1} \right)^{\frac{1}{1}} {}_t = \frac{}{1} Y_t + E_t \left[{}_{t+1} \left(\frac{1 \quad {}_{t+1}}{1} \right)^{\frac{1}{1}} {}_{t+1} \right];$$

$$ {}_t = \frac{Y_t}{Y_t \quad G} + E_t \left[{}_{t+1}^{\;1} \; {}_{t+1} \right];$$

$$\frac{b_t}{R \; {}_t^{\;;t} e^{u_{m;t}}} = \frac{b_{t-1}}{{}_t} + G \quad \left(\frac{b_{t-1}}{b} \right)^{;t} e^{u_{;t}}:$$

Using the notation of FRWZ, this system can be rewritten as

$$E_t f\left(y_{t+1}; y_t; b_t; b_{t-1}; "_{t+1}; "_t; {}_{t+1}; {}_t\right) = 0;$$

where b_t is the only predetermined variable and the remaining non-predetermined variables are stacked in vector $y_t^0 \quad [Y_t; {}_t; {}_t]$. The exogenous shocks appear in vector $"_t^0 \quad [u_{m;t}; u_{;t}]$, and ${}_t^0 \quad [{}_{;t}; {}_{;t}]$ is the vector of Markov-switching parameters. We look for recursive solutions such as

$$b_t = h_t(b_{t-1}; "_t; \;)$$
$$y_t = g_t(b_{t-1}; "_t; \;)$$

perturbed around the non-stochastic zero-in ation steady state $[b; y^0]^0$, where represents the perturbation parameter. Note that in our model the solutions are regime-dependent (h_t and g_t follow the latent Markov process too), while the steady state is not. The stability properties of each solution is governed by parameters of the rst order expansion of the solutions, which reads as follows under regime i

$$b_t \quad b + h_{i;b}(b_{t-1} \quad b) + h_{i;"} "_t + h_{i;} \;; \tag{43}$$

$$y_t \quad y + g_{i;b}(b_{t \quad 1} \quad b) + g_{i;\varepsilon}\,\varepsilon_t + g_{i;} \quad ; \tag{44}$$

for $i = 1;2$. In these expressions we used a matrix notation for the partial derivatives: for example, $g_{i;\varepsilon}$ is a $(3 \quad 2)$ matrix whose rst column is given by the partial derivative of g_i with respect to $u_{m;t}$, and so forth.

The elements in $h_{i;b}$, $h_{i;\varepsilon}$, $h_{i;}$, $g_{i;b}$, $g_{i;\varepsilon}$, $g_{i;}$ are unknown and can be found by exploiting the fact that the derivatives of $E_t f$ are equal to zero. Proposition 1 in FRWZ uses the chain rule to state that the coe cients in $h_{i;b}$ and $g_{i;b}$ can be obtained by solving a system of quadratic polynomial equations that corresponds to equation (A4) in FRWZ. To derive such system, we need to compute the partial derivatives of f

$$f_{ij;\,y_{t+1}} \quad f_{ij;\,y_t} = \begin{bmatrix} \frac{1}{cY} & 1 & 0 & \frac{1}{cY} & {}_{;i} & 0 \\ 0 & \overline{}\,{}_{1} & & \overline{}\,{}_{1} & \overline{}\,{}_{1} & 1 \\ 0 & (1 \quad) & & \frac{1 \quad c}{c^2 Y} & 0 & 1 \\ 0 & 0 & 0 & 0 & (1 \quad {}_{;i})\, b & 0 \end{bmatrix};$$

$$f_{ij;b_t} \quad f_{ij;b_{t \quad 1}} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \bar{b} \quad {}_{;i} & 1 \end{bmatrix}:$$

Note that the derivatives are indexed with ij to indicate that they must be evaluated at the steady state with ${}_t = i$ and ${}_{t+1} = j$ (refer to FRWZ for further details). With these derivatives in hand, we can apply formula (A4) of FRWZ and obtain a set of 8 equations in 8 unknowns

$$0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \bar{b} \quad {}_{;1} \quad 1 \end{bmatrix} + \begin{bmatrix} \frac{1}{cY} & {}_{;1} & 0 \\ \overline{}\,{}_{1} & \overline{}\,{}_{1} & 1 \\ \frac{1 \quad c}{c^2 Y} & 0 & 1 \\ 0 & (1 \quad {}_{;1})\, b & 0 \end{bmatrix} \begin{bmatrix} g_{y;1} \\ g_{;1} \\ g_{;1} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} h_1$$

$$+\, p_{11} \begin{bmatrix} \frac{1}{cY} & 1 & 0 \\ 0 & \overline{}\,{}_{1} & \\ 0 & (1 \quad) & \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} g_{y;1} \\ g_{;1} \\ g_{;1} \end{bmatrix} h_1 + p_{12} \begin{bmatrix} \frac{1}{cY} & 1 & 0 \\ 0 & \overline{}\,{}_{1} & \\ 0 & (1 \quad) & \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} g_{y;2} \\ g_{;2} \\ g_{;2} \end{bmatrix} h_1;$$

$$0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{}{b}\ {}_{;2} \quad 1 \end{bmatrix} + \begin{bmatrix} \frac{1}{cY} & {}_{;2} & 0 \\ \overline{}\ 1 & \frac{}{1} & 1 \\ \frac{1\ \ c}{c^2Y} & 0 & 1 \\ 0 & (1 \quad {}_{;2})\, b & 0 \end{bmatrix} \begin{bmatrix} g_{y;2} \\ g_{;2} \\ g_{;2} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} h_2$$

$$+ \; p_{21} \begin{bmatrix} \frac{1}{cY} & 1 & 0 \\ 0 & \frac{}{1} & \\ 0 & (1 \quad) & \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} g_{y;1} \\ g_{;1} \\ g_{;1} \end{bmatrix} h_2 + p_{22} \begin{bmatrix} \frac{1}{cY} & 1 & 0 \\ 0 & \frac{}{1} & \\ 0 & (1 \quad) & \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} g_{y;2} \\ g_{;2} \\ g_{;2} \end{bmatrix} h_2;$$

where, with a slight abuse of notation, the h_i and $g_{x;i}$ coefficients are the elements of $h_{i;b}$ and $g_{i;b}$ defined above. This system can be simplified by subtracting the third equation from the second, and the seventh from the sixth, to eliminate $g_{;1}$ and $g_{;2}$. We then arrive at

$$0 = \frac{1}{cY} g_{y;1} + \quad {}_{;1} g_{;1} \quad h_1 \;\; p_{11} \quad g_{;1} + \frac{1}{cY} g_{y;1} \quad + p_{12} \quad g_{;2} + \frac{1}{cY} g_{y;2} \quad ; \tag{45}$$

$$0 = g_{;1} \quad \overline{cY}\, g_{y;1} \qquad h_{\ 1} \left(p_{11} g_{;1} + p_{12} g_{;2}\right); \tag{46}$$

$$0 = \;\; h_{\ 1} + b(1 \qquad {}_{;1})\, g_{;1} + \frac{}{b} \;\; {}_{;1} \quad 1; \tag{47}$$

$$0 = \frac{1}{cY} g_{y;2} + \quad {}_{;2} g_{;2} \quad h_2 \;\; p_{21} \quad g_{;1} + \frac{1}{cY} g_{y;1} \quad + p_{22} \quad g_{;2} + \frac{1}{cY} g_{y;2} \quad ; \tag{48}$$

$$0 = g_{;2} \quad \overline{cY}\, g_{y;2} \qquad h_{\ 2} \left(p_{21} g_{;1} + p_{22} g_{;2}\right); \tag{49}$$

$$0 = \;\; h_{\ 2} + b(1 \qquad {}_{;2})\, g_{;2} + \frac{}{b} \;\; {}_{;2} \quad 1: \tag{50}$$

Note that the term appears after exploiting the restriction (42). Finally, these equations can be further combined to obtain

$$0 = g_{;1} \;\; 1 + \quad {}_{;1} \quad p_{11} h_1 (1 + \quad + \quad) + p_{11}^2 \, h_{\ 1}^{\ 2} + (1 \quad p_{11})(1 \quad p_{22}) \;\; h_{\ 1} h_2 g_{;1}$$

$$+ (1 \quad p_{11})\, h_1 g_{;2} \left[p_{11} \, h_{\ 1} + p_{22} \, h_{\ 2} \quad (1 + \quad + \quad)\right];$$

$$0 = g_{;2} \;\; 1 + \quad {}_{;2} \quad p_{22} h_2 (1 + \quad + \quad) + p_{22}^2 \, h_{\ 2}^{\ 2} + (1 \quad p_{11})(1 \quad p_{22}) \;\; h_{\ 1} h_2 g_{;2}$$

$$+ (1 \quad p_{22})\, h_2 g_{;1} \left[p_{11} \, h_{\ 1} + p_{22} \, h_{\ 2} \quad (1 + \quad + \quad)\right];$$

$$g_{;1} = \frac{\frac{1}{} \;\; 1 \quad \frac{}{b} \;\; {}_{;1} \qquad h_1}{b \;\; \frac{1}{} \qquad {}_{;1}};$$

$$g_{;2} = \frac{\frac{1}{} \;\; 1 \quad \frac{}{b} \;\; {}_{;2} \qquad h_2}{b \;\; \frac{1}{} \qquad {}_{;2}};$$

which correspond to equations (18)-(21) in the main text.

As this system cannot be solved using traditional approaches such as the generalised Schur decomposition, we follow FRWZ and adopt the Groebner basis algorithm to find all existing solutions, i.e., all the possible 8-tuples made by coefficients h_1, $g_{y;1}$, $g_{;1}$, $g_{;1}$, h_2, $g_{y;2}$, $g_{;2}$, $g_{;2}$ that satisfy the system of equations.

Note that to characterise the first order expansion of the MSV solution we still have to determine the other coefficients $h_{i;\varepsilon}$, $h_{i;}$, $g_{i;\varepsilon}$, $g_{i;}$ that appear in equations (43) and (44). Fortunately, doing so is an easy task. Proposition 1 in FRWZ shows that one has to solve two separate systems of linear equations corresponding to their equations (A5) and (A6).

A.4 Mean square stability under regime switching

To asses the stability of the MSV solutions when some parameters are allowed to switch, FRWZ use the notion of mean square stability (MSS), which is discussed by Costa et al. (2005) and Farmer et al. (2009).

MSS requires the existence of

$$\lim_{t\to\infty} E_0\left(\left[\hat{y}_t\right]\right); \quad \text{and} \quad \lim_{t\to\infty} E_0\left(\left[\hat{y}_t\right]\left[\hat{y}_t\right]'\right)$$

The MSS condition constrains the values of the autoregressive roots in the state variable policy function weighted by the probability of switching regimes. In our context with one state variable and two regimes, MSS formally states that one solution is stable if an only if all the eigenvalues of the matrix

$$\begin{bmatrix} p_{11} & 1-p_{22} \\ 1-p_{11} & p_{22} \end{bmatrix} \begin{bmatrix} h_1^2 & 0 \\ 0 & h_2^2 \end{bmatrix} = \begin{bmatrix} p_{11}h_1^2 & (1-p_{22})h_2^2 \\ (1-p_{11})h_1^2 & p_{22}h_2^2 \end{bmatrix}$$

are inside the unit circle. The characteristic polynomial of this matrix is

$$z^2 + a_1 z + a_0 = z^2 - \left(p_{11}h_1^2 + p_{22}h_2^2\right) z + \left(p_{11} + p_{22} - 1\right) h_1^2 h_2^2 = 0.$$

As discussed in LaSalle (1986, p. 28), both eigenvalues are inside the unit circle if and only if both the following conditions hold

$$|a_0| < 1$$

$$|a_1| < 1 + a_0;$$

which in our case give

$$(p_{11} + p_{22} \quad 1)\, h_1^2 h_2^2 \ < \ 1;$$
$$p_{11} h_1^2 + p_{22} h_2^2 < 1 + (p_{11} + p_{22} \quad 1)\, h_1^2 h_2^2 :$$

If we assume $p_{11} + p_{22} \quad 1$, the two conditions can be be rewritten as

$$(p_{11} + p_{22} \quad 1)\, h_1^2 h_2^2 < 1;$$
$$p_{11} h_1^2 \ 1 \quad h_2^2 \ + p_{22} h_2^2 \ 1 \quad h_1^2 \ + h_1^2 h_2^2 < 1;$$

which correspond to equations (10) and (11) in the main text.

A.5 Figure 2b: A numerical example for the timid scal deviations

Consider the case with $p_{11} = p_{22} = p < 1$ in Figure 3b. If regime 1 is AM/PF so that $g_{;1} = g_{y;1} = 0$, from system (45)-(50) we can derive the equations

$$0 = \underbrace{\frac{\frac{1}{} \ 1 \quad \frac{}{b} \ _{;1} \qquad h_1}{b \ \frac{1}{} \qquad _{;1}}}_{g_{;1}} \ 1 + \quad _{;1} \quad p_{11} h_1 (1 + \quad + \quad) + p_{11}^2 \, h_1^2 \ ; \tag{51}$$

$$0 = \underbrace{\frac{\frac{1}{} \ 1 \quad \frac{}{b} \ _{;2} \qquad h_2}{b \ \frac{1}{} \qquad _{;2}}}_{g_{;2}} \ 1 + \quad _{;2} \quad p_{22} h_2 (1 + \quad + \quad) + p_{22}^2 \, h_2^2 \ : \tag{52}$$

Call a solution stemming from $g_{;i} = 0$, that therefore depends only on the scal coe cient $_{;1}$, $h_i(\ _{;i})$. Take a passive scal policy in regime 1 with $_{;1} = 0{:}2$ and $p = 0{:}95$. In this case the stable solution is $h_1(\ _{;1})$ that, under our calibration becomes $h_1 = \frac{1}{} \ 1 \quad \frac{}{b} \ _{;1} \ = 0{:}9068$. In order to have MSS, conditions (10) and (11) must hold. Under this case the most stringent one of the two turns out to be equation (11) that becomes $0{:}822p \ 1 \quad h_2^2 \ + 0{:}178p h_2^2 \quad 1 + 0{:}822 h_2^2 < 0$. Then, in order to have MSS the stable solution in regime 2 must satisfy the following: $1{:}0209 < h_2 < 1{:}0209$.[27] If the stable solution in regime 2 is again $h_2(\ _{;2})$ then: $h_2 = \frac{1}{} \ 1 \quad \frac{}{b} \ _{;2} \ \ 2 \ (\ 1{:}0209; 1{:}0209)$. In this

[27] Condition (10) instead gives $1{:}162 < h_2 < 1{:}162$:

case we get a "timid" fiscal deviation for:

$$0.02 < \ _{;2} < 3.93$$

Substituting $h_2 = 1.0209$ in the square bracket in equation (52) we get $_{;2} = 0.955$ which is the lower bound of the correspondent "timid" monetary deviation:

$$0.955 < \ _{;2} < 1$$

Note that these results hold for $_{;1} = 0.2$; obviously, for every given $_{;1}$ we could obtain different $_{;2}$ and $_{;2}$ coefficients.

A.6 The absorbing case

When regime 1 is absorbing ($p_{11} = 1$), MSS requires the eigenvalues of the following matrix to lie inside the unit circle:

$$\begin{bmatrix} h_1^2 & (1 \quad p_{22})h_2^2 \\ 0 & p_{22}h_2^2 \end{bmatrix}$$

The two eigenvalues are equal to h_1^2 and $p_{22}h_2^2$, so that the conditions for MSS are

$$|h_1| < 1; \tag{53}$$

$$|h_2| < \frac{1}{\sqrt{p_{22}}}. \tag{54}$$

Moreover, by plugging $p_{11} = 1$ in equations (18) and (20) we obtain equation (23), that is

$$0 = \frac{\frac{1}{} \ 1 \quad \frac{}{b} \ _{;1} \quad h_1}{b \ \frac{1}{} \qquad _{;1}} \ 1 + \quad _{;1} \quad h_1(1 + \quad + \quad) + \ h_1^2 \ :$$

This equation has three solutions for h_1: one $h_1(\ _{;1})$ that depends on the fiscal coefficient $_{;1}$ (first term), the other two $h_1(\ _{;1})$ that depend on the monetary coefficient $_{;1}$, (square brackets).

When regime 1 is AM/PF then $g_{;1} = g_{y;1} = 0$, since debt has no impact on inflation and output. Using these restrictions, equations (19) and (21) yield equation (52) for the non-absorbing regime.

A.6.1 Figure 2a

Consider an AM/PF regime 1. The stability condition for the absorbing state (53) is the same as under fixed coefficients. As fiscal policy is passive, then

$$\frac{1}{} \quad 1 \quad \frac{}{b} \quad {}_{;1} \quad < 1;$$

that is $(1 \quad)^{\frac{b}{}} < \quad {}_{;1} < (1 + \quad)^{\frac{b}{}}$. Employing our calibration, we have ${}_{;1}$ 2 (0:019; 3:892). The condition for not having another stable solution is that the two solutions in the square bracket of (23) should be outside the unit circle, that is, ${}_{;1} > 1$: monetary policy needs to be active. We now analyse the MSS condition (54) for regime 2, given that regime 1 is AM/PF. To do so, we have to solve the third order equation (52) for h_2, and obtain one solution $h_2(\quad {}_{;2})$ and two solutions $h_2(\quad {}_{;2})$. Let us distinguish two cases according to the stability of the $h_2(\quad {}_{;2})$ solution in regime 2.

A stable $h_2(\quad {}_{;2})$ solution. In this case the solution must satisfy: $\frac{1}{} \quad 1 \quad \frac{}{b} \quad {}_{;2} \quad < \frac{1}{\sqrt{p_{22}}}$; that gives equation (24)

$$\frac{b}{} \quad 1 \quad \sqrt{\overline{p_{22}}} \quad < \quad {}_{;2} < \frac{b}{} \quad 1 + \sqrt{\overline{p_{22}}}$$

which, employing our calibration, returns: ${}_{;2}$ 2 (0:032; 3:952).

To have a unique stable $h_2(\quad {}_{;2})$ solution the other (two $h_2(\quad {}_{;2})$) solutions must be both unstable, which translates into equation (25)

$$ {}_{;2} > \sqrt{\overline{p_{22}}} \quad \frac{1 \quad \sqrt{\overline{p_{22}}} \quad 1 \quad \sqrt{\overline{p_{22}}}}{};$$

that is, ${}_{;2} > 0{:}964$: This first case describes the upper-right zone in Figure 3a.

An unstable $h_2(\quad {}_{;2})$ solution. Under this case $\frac{1}{} \quad 1 \quad \frac{}{b} \quad {}_{;2} \quad > \frac{1}{\sqrt{p_{22}}}$ which corresponds to equation (28). Under our calibration, we have ${}_{;2} < \quad 0{:}032; \quad {}_{;2} > 3{:}952$.

In order to have only one stable solution, the two $h_2(\quad {}_{;2})$ solutions of (52) must be one inside and the other outside the unit circle, which yields equation (29)

$$ {}_{;2} < \sqrt{\overline{p_{22}}} \quad \frac{1 \quad \sqrt{\overline{p_{22}}} \quad 1 \quad \sqrt{\overline{p_{22}}}}{}:$$

Employing our calibration, we get ${}_{;2} < 0{:}964$. Again, monetary policy can be passive, and the more so, the lower p_{22}. This second case describes the lower-left zone in Figure 3a.

As the absorbing regime is AM/PF, we know that the only stable solution for this regime corresponds the scal backing one while, as the value of $_{;1}$ is greater than 1, a stable $h_1(\;_{;1})$ solution does not exist. To have determinacy, there should only be one corresponding stable solution h_2. The threshold values $_{;2}$ and $_{;2}$ for the timid changes in monetary and scal policies de ne the conditions for the existence of a stable solution in the second regime. In particular, starting from an AM/PF absorbing regime, determinacy in regime 2 admits either only timid deviations from AM/PF (upper-right zone, $_{;2} > \;_{;2};\;\; _{;2} > \;_{;2}$) or large deviations in both monetary and scal policy, such that we are de nitely in a PM/AF regime (lower-left zone, $_{;2} < \;_{;2};\;\; _{;2} < \;_{;2}$). More precisely: (i) if scal policy is not too active (if $_{;2} > \;_{;2}$), there exists a scal backing solution, where dynamics are Ricardian in both regimes; (ii) if monetary policy is su ciently passive (if $_{;2} < \;_{;2}$), there exists a scal unbacking solution, with wealth e ect in the second regime.[28] Hence, to have only one solution, either (i) or (ii) should be satis ed. When both are satis ed, we have two solutions as in the white region; when neither is satis ed, we have no stable solutions as in the dark blue region.

A.6.2 Globally balanced policies: analytical results when regime 1 is absorbing

The scal frontier Consider the absorbing case and assume that monetary policy is always active ($_{;1} = \;_{;2} = 1{:}5$). The equation to be solved for h_1 is (23). If $_{;1} > 1$, the roots of the second order equation in the square brackets are out of the unit circle. If in the rst regime there is a PF policy, the equation for the second regime reduces to equation (52). Given that we assumed an active monetary policy even in regime 2, we have global determinacy whenever scal policy is passive (or timidly active): $\frac{b}{} \; 1 \; \frac{}{\sqrt{p_{22}}} < \;_{;2} < \frac{b}{} \; 1 + \frac{}{\sqrt{p_{22}}}$, which corresponds to equation (25) in the text. So we have determinacy for the absorbing regime 1 when $_{;1} > \frac{b}{}(1 \quad)$ and for the non-absorbing regime 2 when $_{;2} > \;_{;2}$. Figure A1 displays what we label the scal frontier : the scal policy combinations above this frontier admit only timid deviations into active scal behaviour in the second regime. The other scal policy combinations in Figure A1, by contrast, do not admit a mean square stable solution $h_i(\;_{;i})$. In these cases, if the $h_1(\;_{;1})$ solution in the rst regime is outside the unit circle ($h_1 = \frac{1}{} \; 1 \quad \frac{}{b}\;_{;1} > 1$), that is if there is an AF policy, then all solutions are explosive, independently from what happens in the second regime.

The monetary frontier Figure A2 displays the monetary frontier in the absorbing case. Take an always passive scal policy, we know that a scal backing solution always exists under the two regimes,

[28] Obviously, there are no wealth e ects in the rst because it is absorbing, so that it does not admit spillovers.

Figure A1: The fiscal policy frontier in the absorbing case.

and thus, determinacy requires all solutions $h_i(\ _{;i})$ to be unstable. The condition for the absorbing state is (23). If in the absorbing regime 1 fiscal policy is passive, then the solution $h_1(\ _{;1})$ is inside the unit circle and $(1\ \ )\frac{b}{} < \ _{1;} < (1+\ \ )\frac{b}{}$. The condition for not having another stable solution is that the two $h_1(\ _{;1})$ solutions should be outside the unit circle, that boils down to the usual $\ _{;1} > 1$: monetary policy needs to be active. As for the non-absorbing regime 2, consider again equation (52). If fiscal policy is passive then to have a unique stable solution the other (two) (52) solutions must be both outside the unit circle, which gives (25): $\ _{;2} > \sqrt{p_{22}} \quad \frac{(1\ \ \sqrt{p_{22}})(1\ \sqrt{p_{22}})}{}$ that is, we have determinacy when $\ _{;2} > \ _{;2}$:

A.6.3 Figure 4b under an absorbing PM regime 1

Suppose now monetary policy is PM (with $\ _{;1} = 0{:}9$) in the first (absorbing) regime and AM (with $\ _{;2} = 1{:}5$) in the second regime. The condition for the absorbing state is, as usual, (23). That for the non-absorbing state is derived from (19) evaluated at $p_{11} = 1$ where, to simplify notation, we define $z = h_2\sqrt{p_{22}}$

$$
\begin{aligned}
0 = {} & g_{;2}\ \ 1 + \ \ _{;2}\ \ z\sqrt{p_{22}}(1 + \ \ + \ \ ) + \ z^2 p_{22} \\
& + (1\ \ p_{22}) g_{;1} z \ \ z \ \ \frac{1}{\sqrt{p_{22}}}(1 + \ \ + \ \ \ \ h_{1})
\end{aligned}
\tag{55}
$$

Figure A2: The monetary policy frontier in the absorbing case.

where $g_{;1}$ and $g_{;2}$ are given, as usual, by equations (20) and (21) in the text. We can re-write the condition for the non-absorbing state as

$$z^3 + b_2 z^2 + b_1 z + b_0 = 0$$

where

$$b_2 = \frac{1 \quad \bar{b} \quad {}_{;2} \; p_{22} + (1 + \quad + \quad) + g_{;1}(1 \quad p_{22}) b \; \frac{1}{} \quad {}_{;2}}{\sqrt{p_{22}}};$$

$$b_1 = \frac{\sqrt{p_{22}}(1 + \quad + \quad) \frac{1}{} \; 1 \quad \bar{b} \quad {}_{;2} + \frac{(1+ \quad {}_{;2})}{\sqrt{p_{22}}} + g_{;1} \frac{(1 \quad p_{22})}{\sqrt{p_{22}}} \; 1 + \quad + \quad h_1 \; b \; \frac{1}{} \quad {}_{;2}}{\sqrt{p_{22}}};$$

$$b_0 = \frac{1 \quad \bar{b} \quad {}_{;2} (1 + \quad {}_{;2})}{2\sqrt{p_{22}}}:$$

The necessary and su cient condition for determinacy is that this cubic equation has exactly one solution inside the unit circle and the other two outside. By proposition C.2 in Woodford (2003), this is the case if and only if either of the following two cases is satis ed:

Case I: $1 + b_2 + b_1 + b_0 < 0$ and $1 + b_2 \quad b_1 + b_0 > 0$;

Case II: $1 + b_2 + b_1 + b_0 > 0$, $1 + b_2 \quad b_1 + b_0 < 0$, and $b_0^2 \quad b_0 b_2 + b_1 \quad 1 > 0$ or $|b_2| > 3$.

Let us study now how determinacy varies according to the scal policy undertaken in regime 1.

AF in the absorbing regime 1. Consider the case of an AF policy in the absorbing regime: regime 1 will have a PM/AF mix hence a unique stable solution. In this case the $h_1(\ _{;1})$ solutions in (23) should have a root inside and one outside the unit circle while the $h_1(\ _{;1})$ solution, being AF, is outside. The monetary coe cient $\ _{;1} = 0{:}9$ generates a stable solution $h_1 = 0{:}94343$ (and an unstable one, that we discard, $h_1 = 1{:}1534$).

Studying the necessary and su cient conditions for determinacy, we nd that, under these parameters, Case I is never satis ed (since the second inequality never holds) while Case II is. In particular, the condition to have exactly one solution inside the unit circle for regime 2 (from the rst condition in Case II) reads

$$\ _{;1} > \frac{b}{}(1 \quad h_1) \frac{1 + \ _{;2} \, \mathrm{p}\overline{p_{22}}(1 + \quad + \quad) + \ \mathrm{p}_{22}}{(1 \quad p_{22}) \quad \mathrm{p}\frac{1}{\overline{p_{22}}}(1 + \quad + \quad h_1) \ \frac{1}{} \ _{;2}} \ \frac{1}{} \quad _{;1} \ \frac{b}{} \ \mathrm{p}\frac{1}{\overline{p_{22}}} \ 1 + \frac{}{b} \ _{;2} \ ; \tag{56}$$

that is represented by a negative sloped line in the space $(\ _{;1};\ \ _{;2})$ and that depends, among other things, on h_1. Note that (56) corresponds to (55) for $z = 1$.

Hence there is a unique stable solution when $\ _{;1}$ is above this line for $h_1 = 0{:}94343$. As a result, when the rst regime is PM/AF we have a global determinate equilibrium in the hatched area of Figure A3.

Figure A3: Stability for an absorbing PM/AF regime 1.

PF in the absorbing regime 1. When the rst regime is PM/PF, the two $h_1(\;_{;1})$ solutions are one inside ($h_1 = 0{:}94343$) and the other outside while the $h_1(\;_{;1})$ solution is inside with $g_{;1} = 0$.

Now we have to consider two areas, one for each $h_1 < 1$ in regime 1. The solution $h_1 = 0{:}94343$, returns the same negative sloped straight line as before. So, again, there is a unique stable solution for regime 2 when $\;_{;1}$ is above this line. Hence, when the rst regime is PM/PF we have a global determinate equilibrium in the hatched area of Figure A4.

Figure A4: Stability for absorbing PM/PF regime 1: stable monetary solution case.

When we consider the other stable solution, with $g_{;1} = 0$, equation (55) reduces to

$$0 = g_{2;\;;b} \quad 1 + \quad _{;2} \quad z^{p}\overline{p_{22}}\,(1 + \quad + \quad) + \quad z^{2} p_{22} \; :$$

Hence, we have just one stable solution if and only if the regime 2 is AM/PF (in this case there are 2 $h_2(\;_{;2})$ solutions outside and 1 $h_2(\;_{;2})$ solution inside) and the area characterised by just one stable solution is the hatched area in Figure A5.

Overlapping these two gures (A4 and A5) we get the areas with just one stable solution when scal policy in regime 1 is passive: the two triangular areas in Figure A6.

Putting everything together. Putting together Figures A3 and A6, one obtains A7, which is the counterpart of Figure 2b for the case of absorbing PM regime 1.

Figure A5: Stability for absorbing PM/PF regime 1: stable scal solution case.

Figure A6: Stability for absorbing PM/PF regime 1: complete case.

Figure A7: The scal policy frontier with substantial deviations in monetary policy and absorbing regime 1 with PM.

Notes: Light blue: unique solution; white: indeterminacy; dark blue: explosiveness.

A.7 Both regimes stable

In this section we want to show how our results change when the concept of both regimes stable (BRS) is used in place of the MSS concept that we used throughout the paper.[29] Note that MSS allows temporarily explosive paths while BRS does not. More specifically, a solution is BRS if all h_i lie inside the unit circle, for $i = 1, 2$.

Figure A8 shows the monetary policy frontier (MPF): the figure reproduces Figure 1a in the main text, with the addition of sparse green hatch lines that indicate the area where a single BRS solution exists. The edge of the this area can be interpreted as the MPF under BRS. As you can see, the MPF shifts outwards if BRS is considered: the MPF under MSS, which encloses the policy mixes characterised by a unique stable (Ricardian) solution, lies inside the new MPF under BRS. In the area between the two frontiers, policy mixes are characterized by the co-existence of two MSS solutions: one solution is Ricardian, while the second solution is non-Ricardian and explosive in one regime (i.e., either h_1 or h_2 is greater than unity). Indeterminacy then arises under MSS but not under BRS as the second solution is not stable in both regimes. Hence this additional area is included in the determinacy region that therefore gets larger. Outside the green-hatched area, the policy mixes give rise to two stable solutions, under both stability concepts (one of these solutions is Ricardian). These parametrizations are therefore indeterminate.

Figure A9 depicts the fiscal policy frontier (FPF) for timid deviations into passive monetary policy. Policy mixes in the green-hatched area are characterized by single stable solution under both MSS and BRS. This solution, which is Ricardian, becomes explosive in one of the two regimes once fiscal policy deviates timidly into active fiscal territory (red-hatched area), but MSS is preserved. As no other stable solutions exist, we have determinacy under MSS but not under BRS. In other words, BRS rules out any flexibility in fiscal policy and the FPF coincides with the edges of the usual conditions for passive fiscal policy by Leeper (1991).

Figure A10 shows how Figure 2b, which depicts determinacy with substantial deviations in monetary policy, changes under BRS. Note that, even in this case, the FPF is not relevant anymore. A timid fiscal policy mix (e.g, consider the parametrization ${}_{;1} = 0.2$, ${}_{;2} = 0$) is characterized by one stable non-Ricardian solution and one MSS Ricardian solution that is explosive in one regime. The equilibrium is than indeterminate under MSS but determinate under BRS. In this case, therefore, employing this alternative stability concept, timid fiscal policy produces wealth effects (as in Chung et al., 2007) and the fiscal theory is always at work.

[29] Refer to Foerster (2016) for a discussion of BRS.

Starting from a AM/PF regime 1, a su cient condition to have a unique stable solution and wealth e ects under MSS is to have substantial variation in both monetary and scal policy in regime 2; under BRS just a substantial variation in monetary policy in regime 2 is needed while scal policy should be (even timidly) active.

Figure A8: The monetary policy frontier under MSS and BRS.

Notes: Light blue: unique MSS solution; white: multiple MSS solutions; dark blue: no MSS solutions; green-hatching: unique BRS solution.

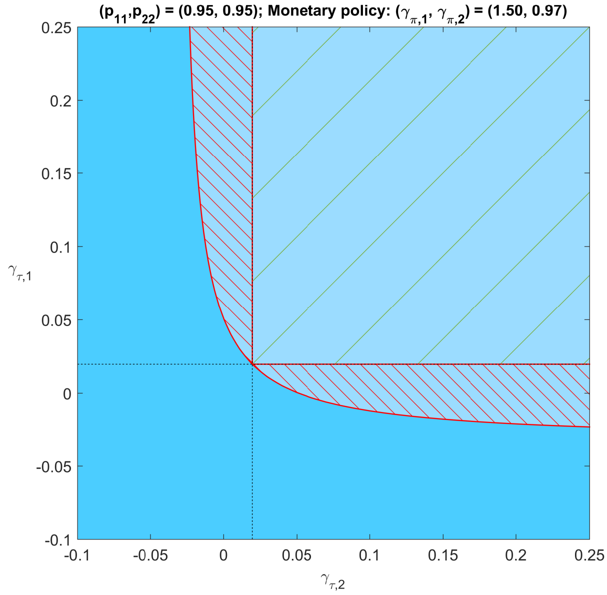

Figure A9: The fiscal policy frontier under MSS and BRS, for timid deviations in monetary policy.

Notes: Light blue: unique MSS solution; white: multiple MSS solutions; dark blue: no MSS solutions; green-hatching: unique BRS solution.

Figure A10: The fiscal policy frontier under MSS and BRS, for substantial deviations in monetary policy.

Notes: Light blue: unique MSS solution; white: multiple MSS solutions; dark blue: no MSS solutions; green-hatching: unique BRS solution.